A Simulation on the Public Good Provision under Various Taxation Systems

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1. Introduction

1. The Examination of *Optimal Taxation* under *General Equilibrium* (GE) Model is the Purpose of this Paper.
2. On the One Hand, the Traditional Argument in the Textbook Examines the Optimal Taxation under *Partial Equilibrium* Model: e.g. the *Commodity Tax* is Inferior to *Poll Tax*, since the Former Gives Rise to *Dead Weight Loss*, while the Latter does not.
3. In the *Ramsey Rule* Argument the Government Derives the Optimal Commodity Tax Rates in Order to Achieve the Fixed Amount of Tax Revenue.
4. In these Arguments the Reason why the Tax Revenue is Necessary is not Specified.
5. On the Other Hand, in the Textbook, the *Optimal Public Good Provision* is one of the Main Roles of the Government. In this Provision, the *Lindahl Mechanism* has been Adopted.
6. In this Paper, the Payment of the Economic Agents is Defined as the *Lindahl Tax* and Compares it with Other Taxations to Sustain the Optimal Level of Public Good.
7. The Lindahl Mechanism is Utilized under the GE Framework.

2. Economy with Public Good and Walras - Lindahl Mechanism: *Specified* GE Model

2.1 Production Side: 3 Commodities, *x*, *y*, *z*, Produced by Labor, *L*, and Capital, *K*, w_L: Wage Rate, w_K: Rental Price of Capital

1. *x*, *y*: Private Consumption Goods, p_x: Price of x, p_y: Price of y,
2. *z*: *Public Good*, p_z: Price of z,
3. L_e: Social Endowment of Labor, K_e: Social Endowment of Capital
4. Sector 1: $y = f_1 = L_1^{a1} K_1^{b1}$, with $a_1 + b_1 < 1$: *Decreasing* Returns to Scale, Owned by *Entrepreneur* 1
   Behavior: *Max* $\pi_1 = p_y y - w_L L_1 - w_K K_1$
   (L_1^D: Demand for Labor, K_1^D: Demand for Capital, y^S: Supply of y)
5. Sector 2: $x = f_2 = L_2^{a2} K_2^{b2}$, with $a_2 + b_2 < 1$: *Decreasing* Returns to Scale, Owned by *Entrepreneur* 2
   Behavior: *Max* $\pi_2 = p_x x - w_L L_2 - w_K K_2$
   (L_2^D: Demand for Labor, K_2^D: Demand for Capital, $x^{S:}$ Supply of x)
6. Sector 3: $z = f_3 = L_3^{a3} K_3^{b3}$, with $a_3 + b_3 = 1$: *Constant* Returns to Scale, Owned by the *Government*, for Simplicity
   Behavior: *Min* $w_L L_3 + w_K K_3$ s.t $z = L_3^{a3} K_3^{b3}$: Constant
   (L_3^D: Demand for Labor, K_3^D: Demand for Capital, *Given z*)

- 2.2 Consumer Side: 4 Consumers of x, y, z

1. Household L, the (Aggregate) *Worker*: Income $m_L = w_L \alpha_L L_e + w_K \beta_L K_e$
2. Household K, the (Aggregate) *Capitatist*: Income $m_K = w_L \alpha_K L_e + w_K \beta_K K_e$
3. Household 1, *Entrepreneur* 1: Income $m_1 = \pi_1$
4. Household 2, *Entrepreneur* 2: Income $m_2 = \pi_2$
5. They Have the Same *CES Utility Function*,

$$u[y, x, z] = (\gamma_y y^k + \gamma_x x^k + \gamma_z z^k)^{1/k}$$

6. Their Behavior:

$$\text{Max } u[y, x, z] \text{ s.t. } p_y y + p_x x + \theta_j p_z z = m_j \ (j = L, K, 1, 2) \qquad (1)$$

where θ_j is the Burden Share of the Household j for the Public Good ($j = L, K$, 1, 2).

7. Demand Functions for the Commodities:

Household L: y_L^D, x_L^D, z_L^D

Household K: y_K^D, x_K^D, z_K^D

Household 1: $y_{E1}^D, x_{E1}^D, z_{E1}^D$

Household 2: $y_{E2}^D, x_{E2}^D, z_{E2}^D$

3. General Equilibrium with Public Good: Walras-Lindahl Equilibrium with Lindahl Tax

3.1 Definition of *GE with Public Good*

$y_L^D + y_K^D + y_{E1}^D + y_{E2}^D = y^S$ (2)

$x_L^D + x_K^D + x_{E1}^D + x_{E2}^D = x^S$ (3)

$z_L^D = z_K^D = z_{E1}^D = z_{E2}^D = z$ (4)

$L_1^D + L_2^D + L_3^D = L_e$ (5)

$K_1^D + K_2^D + K_3^D = K_e$ (6)

3.2 *Specification* of Parameters and Computation of GE by the *Newton Method*

$f_1=L_1^{1/6}K_1^{1/5}, f_2=L_2^{1/4}K_2^{1/3}, f_3=L_3^{1/3}K_3^{2/3}, L_e=100, K_e=50,$

$m_L=w_L L_e, m_K=w_K K_e, u[y, x, z] = (y^{1/2}+x^{1/2}+z^{1/2})^2$

$p_x=14.153545624140019458, p_y=19.279639252690992305,$

$w_K=3.8103703458622201692, \theta_1=0.061932787239262383167,$ (7)

$\theta_L=0.32500153599364843437, \theta_K=0.55737371673138213739,$

$z= 56.385555371786405 78, p_z= 4.6104790191979988818, w_L=1$

3.3 *Walras-Lindahl Mechanism* where t: Time. *"Global Stability"*

When 0<k<1

$$
\begin{aligned}
&\mathrm{d}p_y[t]/\mathrm{d}t = y_L{}^{\mathrm{D}} + y_K{}^{\mathrm{D}} + y_{E1}{}^{\mathrm{D}} + y_{E2}{}^{\mathrm{D}} - y^{\mathrm{S}} \\
&\mathrm{d}p_x[t]/\mathrm{d}t = x_L{}^{\mathrm{D}} + x_K{}^{\mathrm{D}} + x_{E1}{}^{\mathrm{D}} + x_{E2}{}^{\mathrm{D}} - x^{\mathrm{S}} \\
&\mathrm{d}w_K[t]/\mathrm{d}t = K_1{}^{\mathrm{D}} + K_2{}^{\mathrm{D}} + K_3{}^{\mathrm{D}} - K_e \\
&\mathrm{d}\theta_L[t]/\mathrm{d}t = z_L{}^{\mathrm{D}} - (z_L{}^{\mathrm{D}} + z_K{}^{\mathrm{D}} + z_{E1}{}^{\mathrm{D}} + z_{E2}{}^{\mathrm{D}})/4 \qquad (8) \\
&\mathrm{d}\theta_K[t]/\mathrm{d}t = z_K{}^{\mathrm{D}} - (z_L{}^{\mathrm{D}} + z_K{}^{\mathrm{D}} + z_{E1}{}^{\mathrm{D}} + z_{E2}{}^{\mathrm{D}})/4 \\
&\mathrm{d}\theta_1[t]/\mathrm{d}t = z_{E1}{}^{\mathrm{D}} - (z_L{}^{\mathrm{D}} + z_K{}^{\mathrm{D}} + z_{E1}{}^{\mathrm{D}} + z_{E2}{}^{\mathrm{D}})/4 \\
&\mathrm{d}z[t]/\mathrm{d}t = z_{E2}{}^{\mathrm{D}} - z[t]
\end{aligned}
$$

3.4 GE Incomes after the Deduction of *Lindahl tax*, GE utility levels, and Gini Coefficients before and after the Lindahl tax

$$
\begin{aligned}
&\{m_L{}^* = 15.5111639395550368,\ m_K{}^* = 45.6211818594757041 5, \\
&\qquad m_{E1}{}^* = 0.5632765368126488,\ m_{E2}{}^* = 0.4554687305937479 6\} \\
&\{u_L{}^* = 78.9894717160154965 0,\ u_K{}^* = 97.4813312057098663 6, \\
&\qquad u_{E1}{}^* = 60.399850877029615 66,\ u_{E2}{}^* = 59.989085914101633 4\} \\
&Gini^{L0} = 0.47350388823249760 43,\ Gini^{L} = 0.60515855230325193 2 \qquad (9)
\end{aligned}
$$

4. GE with Public Good: *Income Tax* Provision of Public Good

4.1 Given z^O= 56.385555371786405 78, All the Consumers Maximize Utility Subject To Income Constraint:

$$\text{Max } u[y, x, z^O] \text{ s.t. } p_y y + p_x x = (1-\tau_I) m_j \quad (j = L, K, 1, 2) \qquad (10)$$

Conditin (4), $z_L^D = z_K^D = z_{E1}^D = z_{E2}^D = z$, Is Replaced by

$$p_z z = t_I (w_L L_e + w_K K_e + \pi_1 + \pi_2) \qquad (11)$$

4.2 By the Newton Method, We Have GE:

$$p_x^I = 14.153545624140019458, \; p_y^I = 19.279639252690992305,$$
$$w_K^I = 3.810370345862220 1692, \; p_z^I = 4.610479019197998 8818, \qquad (12)$$
$$\tau_I = 0.807053427700251179 50$$

4.3 GE Incomes after the Deduction of *Lindahl tax*, GE utility levels, and Gini Coefficients before and after the Lindahl tax

$\{m_L^{*I}$=19.294657229974882050, m_K^{*I}= 36.759894871336189 14,
m_{E1}^{*I}=3.2151823443070032 83, m_{E2}^{*I}=2.8813477376881461 98$\}$

$\{u_L^{*I}$=81.840387820109724 44, u_K^{*I}=92.761339229866624 03,
u_{E1}^{*I}=66.205400915364716 78, u_{E2}^{*I}= 65.661741232324414 54$\}$

$$u_L^{*I} + u_K^{*I} + u_{E1}^{*I} + u_{E2}^{*I} > u_L^* + u_K^* + u_{E1}^* + u_{E2}^*$$
$$Gini^I = 0.47350388823249760 43 = Gini^{L0} < Gini^L$$

Income Tax Is Superior to Lindahl Tax

5. GE with Public Good: *Proportional Commodity Tax* Provision of Public Good

5.1 *Given* z^O= 56.38555537178640578, All the Consumers Maximize Utility Subject To Income Constraint:

$$\text{Max } u[y, x, z^O] \text{ s.t. } (1-\tau_C)p_y y+(1-\tau_C)p_x x=m_j \quad (j=L, K, 1, 2) \qquad (14)$$

Conditin (4), $z_L{}^D=z_K{}^D=z_{E1}{}^D=z_{E2}{}^D=z$, Is Replaced by

$$p_z z=\tau_C\, p_y\, (y_L{}^D+y_K{}^D+y_{E1}{}^D+y_{E2}{}^D) + \tau_C\, p_x\, (x_L{}^D+x_K{}^D+x_{E1}{}^D+x_{E2}{}^D) \qquad (15)$$

5.2 By the Newton Method, We Have GE:

$p_x{}^C$=14.153545624140019458=$p_x{}^l$, $p_y{}^C$=19.279639252690992305=$p_y{}^l$,
$w_K{}^C$=3.8103703458622201692=$w_K{}^l$, $p_z{}^C$=4.610479019197998818=$p_z{}^l$,
τ_C= 4.18278

5.3 GE Incomes after the Deduction of *Lindahl tax*, GE utility levels, and *Gini Coefficients* before and after the Lindahl tax

$\{m_L{}^{*C}=m_L{}^{*l}, m_K{}^{*C}= m_K{}^{*l}, m_{E1}{}^{*C}=m_{E1}{}^{*l}, m_{E2}{}^{*C}=m_{E2}{}^{*l}\}$

$\{u_L{}^{*l}= u_L{}^{*l}, u_K{}^{*l}= u_K{}^{*l}, u_{E1}{}^{*l}=u_{E1}{}^{*l}, u_{E2}{}^{*l}= u_{E2}{}^{*l}\}$

$u_L{}^{*C}+u_K{}^{*C} +u_{E1}{}^{*C}+u_{E2}{}^{*C}=u_L{}^{*l}+u_K{}^{*l} +u_{E1}{}^{*l}+u_{E2}{}^{*l}> u_L{}^*+u_K{}^*+u_{E1}{}^*+u_{E2}{}^*$

$Gini^C=Gini^l$ =0.4735038882324976043=$Gini^{L0}< Gini^L$

Proportional Commodity Tax Is Superior to Lindahl Tax

6. GE with Public Good: *Poll Tax* Provision of Public Good

6.1 *Given* z^O= 56.38555537178640578, All the Consumers Maximize Utility Subject To Income Constraint, where T is Total Tax:

$$\text{Max } u[y, x, z^O] \text{ s.t. } p_y y + p_x x = (m_j - T/4) \quad (j=L, K, 1, 2) \tag{16}$$

Conditin (4), $z_L^D = z_K^D = z_{E1}^D = z_{E2}^D = z$, Is Replaced by

$$p_z z = T \tag{17}$$

6.2 By the Newton Method, We Have GE Prices and Tax:

$p_x^T = p_x^C = p_x^I$, $p_y^T = p_y^C = p_y^I$,

$w_K^T = w_K^C = w_K^I$, $p_z^T = p_z^C = p_z^I$,

T= 259.96442078998567379

6.3 *The Poll Tax cannot Sustain* z^O, *since the Income after the Poll Tax is Negative for Entrepreneurs 1 and 2*.

$\pi_1 - T/4 = -48.327516381250892247$,

$\pi_2 - T/4 = -50.05770833356105865$

Margaret Thatcher Government's Collapse in 1990, when She Introduced the Poll Tax

7. *Robustness* of the Previous Specified GE Model

7.1 The Modification of the Parameters

$f_1' = L_1^{2/3} K_1^{1/8}$, $f_2' = L_2^{1/2} K_2^{1/3}$, $f_3' = L_3^{3/5} K_3^{2/5}$, $L_e = 100$, $K_e = 50$,
$m_L' = (2/3) w_L L_e + (1/5) w_K K_e$, $m_K' = (1/3) w_L L_e + (4/5) w_K K_e$,
$u'[y, x, z] = (y^{1/2} + x^{1/2} + (1/100) z^{1/2})^2$

7.2 GE with Public Good: Walras-Lindahl Equilibrium with Lindahl Tax

The Optimal Public Level: $z^{O\prime} = 0.038835 < z^{O} = 56.3855$

GE Prices and Lindahl Tax Rates:
$p_y' = 4.15817$, $p_x' = 3.98483$, $p_z' = 1.78913$, $w_K' = 0.795961$, $\theta_1' = 0.169305$,
$\theta_L' = 0.34945$, $\theta_K' = 0.326561$

The Gini Coefficients before and after the Lindahl Tax and the Bentham-Type Utilitarian Social Utility Level

$Gini^{L0\prime} = 0.3309739379674684046$, $Gini^{L\prime} = 0.3310327896068688795$
$u_L^{*\prime} + u_K^{*\prime} + u_{E1}^{*\prime} + u_{E2}^{*\prime} = 84.5372799264331946$

7.3 GE with Public Good: *Income Tax* Provision of Public Good

$Gini^{I}{}' = 0.33097393796746846 = Gini^{L0}{}' < Gini^{L}{}'$

$u_L{}^{*I}{}' + u_K{}^{*I}{}' + u_{E1}{}^{*I}{}' + u_{E2}{}^{*I}{}' =$

$84.53727972251664468 > u_L{}^{*}{}' + u_K{}^{*}{}' + u_{E1}{}^{*}{}' + u_{E2}{}^{*}{}'$

Income Tax is more Desirable than the Lindahl Tax

7.4 GE with Public Good: *Proportional Commodity Tax* Provision of Public Good

Exactly the Same Result as in the Income Tax Case

7.5 GE with Public Good: *Poll Tax* Provision of Public Good

$z^{O}{}' = 0.038835 < z^{O} = 56.3855$

$\downarrow$

$T' = 0.06948091519548893442 3 < T = 259.96442078998567379$

Every Member has Positive Income After the Poll Tax

$Gini^{P}{}' = 0.33110773130887710 57 > Gini^{L}{}' > Gini^{I}{}'$

$u_L{}^{*P}{}' + u_K{}^{*P}{}' + u_{E1}{}^{*P}{}' + u_{E2}{}^{*P}{}' = 84.5372757638753862 4 < u_L{}^{*}{}' + u_K{}^{*}{}' + u_{E1}{}^{*}{}' + u_{E2}{}^{*}{}' <$

$u_L{}^{*I}{}' + u_K{}^{*I}{}' + u_{E1}{}^{*I}{}' + u_{E2}{}^{*I}{}'$

The Income Tax is the Best Taxation and the Poll Tax is the Worst Taxation.

8. Simulation *when $0<k<1$*

8.1 *Basic Simulation* for "100 Random Selection Cases" and Result

I. 100 Tuples of Parameters for $\{a_1, b_1, a_2, b_2, a_3, b_3, L_e, K_e, \alpha_L, \alpha_K, \beta_L, \beta_K, k, \gamma_y, \gamma_x, \gamma_z\}$, Selected Randomly, where $a_i+b_i<1$, $i=1, 2$, $a_3+b_3=1$, $\alpha_L+\alpha_K=1$, $\beta_L+\beta_K=1$, and $0<k<1$,

1. a_i, b_j and α_L etc. and k are Expressed by n/m for Integers n and m which Belongs to [1, 10],
2. L_e and K_e are Integers Belonging to [1, 1000],
3. γ_y, γ_x, γ_z are Integers Belonging to [1, 10]

II. The Computation of GE prices, Tax Rates, Utilities, and Incomes by the Newton Method

Among 100 Simulations *Only 65 Cases Satisfy Required 22 Equilibrium Conditions*.

↑

Fixed Initial Position on the Newton Method

III. Among the 65 Cases, 59 Cases (90%) Satisfied

$Gini^L > Gini^I$ and $u_L^*+u_K^*+u_{E1}^*+u_{E2}^* < u_L^{*I}+u_K^{*I}+u_{E1}^{*I}+u_{E2}^{*I}$.

Income Tax (and Proportional Commodity) Tax is More Desirable than Lindahl Tax

8.2 50 Times Repetition of *the Basic Simulation*

Shares (Ratio) of the Cases which Satisfied

1. $Gini^L > Gini^I$
2. $u_L^* + u_K^* + u_{E1}^* + u_{E2}^* < u_L^{*I} + u_K^{*I} + u_{E1}^{*I} + u_{E2}^{*I}$

to the "Successful" Simulations.

{0.913793, 0.934426, 0.939394, 0.896552, 0.936508, 0.955882, 0.921875, 0.875, 0.924528, 0.923077, 0.983871, 0.898551, 0.903226, 0.964286, 1, 0.95082, 0.870968, 0.940299, 0.919355, 0.931034, 0.963636, 0.919355, 0.916667, 0.962963, 0.916667, 0.890909, 0.980769, 0.935484, 0.984848, 0.958904, 0.916667, 0.9, 0.876923, 0.983333, 0.965517, 0.916667, 0.955224, 0.9375, 0.9375, 0.861538, 0.984375, 0.82, 0.936508, 0.885714, 0.915254, 0.901639, 0.857143, 0.936508, 0.935484, 0.919355}

More than 90% of the "Successful" Simulations Guaranteed Income Tax (and Proportional Commodity Tax) is More Desirable than Lindahl Tax when 0<k<1

9. Simulation when $-10<k<0$

9.1 *Completely Different Conclusion When k<0*

When $k=-2$ (a_1=1/8, b_1 =4/5, a_2=5/6, b_2 =1/7, a_2=2/7, b_3 =5/7, L_e=348, K_e=878, α_L=1/2, α_K=1/2, β_L=4/9, and β_K=5/9, $k=-2$, γ_y=13, γ_x=9, and γ_z=4), We Have

1. $Gini^L<Gini^l$ (18)
2. $u_L^*+u_K^*+u_{E1}^*+u_{E2}^*>u_L^{*l}+u_K^{*l}+u_{E1}^{*l}+u_{E2}^{*l}$ (19)

9.2 *Basic Simulation* for "100 Random Selection Cases" and Result when $-10<k<0$

I. 100 Tuples of Parameters for $\{a_1, b_1, a_2, b_2, a_3, b_3, L_e, K_e, \alpha_L, \alpha_K, \beta_L, \beta_K, k, \gamma_y, \gamma_x, \gamma_z\}$

1'. a_i, b_j and α_L etc. are Expressed by *n/m for Integers n and m* which Belongs to [1, 10] and k is *Integer*, Randomly Selected from [– 10, – 1]

II. The Computation of GE prices, Tax Rates, Utilities, and Incomes by the Newton Method: Among 100 Simulations *Only 33 Cases Satisfy Required 22 Equilibrium Conditions* ←Selection of *Fixed Initial Position*

III. Among the 33 Cases, 33 Cases (100%) Satisfied (18) and (19)

Lindahl Tax is More Desirable than Income Tax (and Proportional Commodity Tax)

9.3 50 Times Repetition of the *Basic Simulation*

Numbers of the "Successful" Cases among 100 cases which Satisfied Required 22 Conditions

{28, 37, 34, 35, 32, 29, 26, 26, 27, 34, 28, 34, 30, 26, 28, 25, 32, 33, 27, 31, 25, 29, 27, 36, 28, 32, 23, 32, 25, 37, 33, 33, 28, 21, 29, 25, 23, 30, 31, 33, 34, 25, 32, 33, 34, 31, 30, 32, 28, 32}

Shares (Ratio) of the Cases which Satisfied (18) and (19) to the "Successful" Simulations.

{0.928571, 0.972973, 0.970588, 1, 0.9375, 0.931034, 1, 1, 1, 0.970588, 1, 1, 1, 0.961538, 0.964286, 0.96, 1, 0.969697, 1, 1, 1, 1, 0.962963, 0.944444, 0.964286, 0.96875, 0.913043, 0.9375, 1, 0.945946, 1, 1, 0.964286, 1, 1, 0.92, 1, 0.966667, 0.967742, 1, 0.970588, 1, 1, 1, 0.970588, 1, 0.966667, 0.96875, 0.928571, 0.96875}

More than 90% of the "Successful" Simulations Guaranteed Lindahl Tax is More Desirable than Income Tax (and Proportional Commodity Tax) when $-10<k<0$.

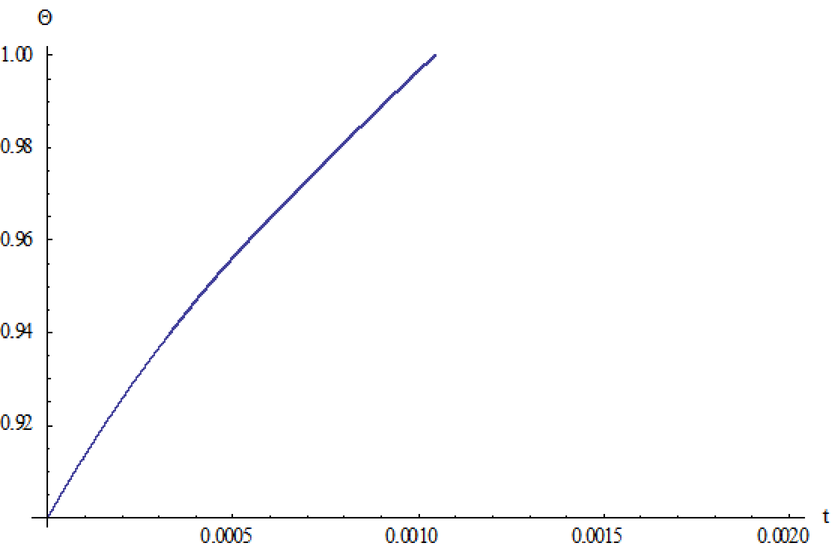
10. Stability Analysis

10.1 Stability of Walras-Lindahl Mechanism

I. (8) Is *Globally Stable* when 0<k<1

II. (8) is *Locally Stable* when k<0

Example:



Parameters

a_1=1/8, b_1 =4/5, a_2=5/6, b_2 =1/7,

a_2=2/7, b_3 =5/7, L_e=348, K_e=878,

α_L=1/2, α_K=1/2, β_L=4/9, β_K=5/9,

k=-2, γ_y=13, γ_x=9, and γ_z=4 (20)

Initial Positions

$\theta_1[0]=5/10$, $\theta_L[0]=1/10$, $\theta_K[0]=3/10$

↓

As t →0.0010

$\Theta[t]=\theta_1[t]+\theta_L[t]+\theta_K[t]\rightarrow 1$: *Unstable*

- *10.2 Walrasian Tatonnement Process to Compute the Rate of Income Tax*

$$d p_y[t]/dt = y_L^D + y_K^D + y_{E1}^D + y_{E2}^D - y^S$$
$$d p_x[t]/dt = x_L^D + x_K^D + x_{E1}^D + x_{E2}^D - x^S \qquad (21)$$
$$d w_K[t]/dt = K_1^D + K_2^D + K_3^D - K_e$$
$$d \tau_I[t]/dt = p_z z - \tau_I (w_L L_e + w_K K_e + \pi_1 + \pi_2)$$

where $p_z z = w_L L_3^D + w_K K_3^D \leftarrow$ *Constant Returns to Scale*

I. (21) is *Globally Stable* when $0<k<1$.

II. (21) is *Locally Unstable*, however, when $k<0$. The set of *Eigen-values* on the Jacobian matrix for (21) is

{-24237.6, 3316.82, -652.992, -202.928}.

Conclusions

k: Parameter on *CES Utility Function*, $u[y, x, z] = (\gamma_y y^k + \gamma_x x^k + \gamma_z z^k)^{1/k}$

1. *When 0<k<1*, There Exists No General Equilibrium for the Poll Tax Case under some *Specification* of Parameters.

2. *When 0<k<1*, *Specifying* Parameters on Production and Utility Functions and Initial Endowments Randomly, We showed that the Income Tax (and Proportional Commodity Tax) Tend to be More Desirable than the Lindahl Tax from the *Fairness* and *Efficiency* Viewpoints with *High possibility of Non-Existence for Poll Tax General Equilibrium.*

3. *When k<0*, however, Specifying Parameters on Production and Utility Functions and Initial Endowments Randomly, We showed that the Lindahl Tax Tends to be More Desirable than the Income Tax (and Proportional Commodity Tax) from the Two Viewpoints.

4. Constructing the Walrasian Tatonnement Process to Compute the Rate of Income Tax, We Attempted the Stability Analysis. *When 0<k<1*, This Process is *Globally Stable* and We Can Compute the Rate of Tax *with Small Amount of Information*, while It Is *Locally Unstable when k<0*. Thus, We May Conclude that *the Income Tax* (and *Proportional Commodity Tax*) is More Desirable than *the Lindahl Tax*.

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