Game-theoretic study of electricity market mechanisms. Alexander Vasin Lomonosov Moscow State University Information Technology and Quantitative Management (ITQM 2014)

A typical structure of electricity market is oligopoly. Consumers usually do not play an active role in electricity auctions. Their behavior corresponds to a known demand function with a low elasticity. So an important problem for such markets is limitation of large producers' market power. Splitting of the electricity market into small companies is a bad way to deal with the problem because of the scale effect and the reliability requirements. (Stoft, 2006, estimates the scale effect as valuable for capacities till 3 GVt).

Another way is to design such mechanism that its equilibrium state coincides with or is maximally close to the Walrasian equilibrium – the optimal state of the market according to the Welfare theorem (Debreu, 1954). The literature on the markets of homogeneous goods (Amir, 1996, Amir & Lambson, 2000, Ausubel & Cramton, 2004, Allen & Hellwing, 1986, Vives, 1986, Durakovich, Vasina, & Vasin, 2003, Shamanaev, 2010, and many others) models different mechanisms as strategic games where producers are the players, and examines Nash equilibrium or its refinement (SPR, SFE) as behavior model. Other desirable properties of the market: existence of the Nash equilibrium in dominating strategies; the strategies can be determined proceeding from the private information of an agent and reveal his real characteristics ("revelation principle").

The main component of any wholesale electricity market is a day ahead market (DAM). Its typical design is the uniform price auction where a producer's bid determines the supplied capacity depending on the price. The market price corresponds to the intersection of the total supply function with the demand function. The real auctions differ in the rules for acceptable bids. Russian DAM accepts bids with at most 3 steps, a different bid for every hour of the next day, while the market of England and Welsh permitted up to 48 steps, but a unique bid for the whole day. Theoretical analysis of uniform price auctions includes following directions.

1) many papers (see Novshek (1985), Kukushkin (1994), Amir (1996), Amir & Lambson (2000) and so on) study the Cournot auction where each seller proposes a fixed amount of the good. (Vasin, Vasina, Ruleva 2007) prove existence of the unique Nash equilibrium under non-decreasing demand elasticity and marginal costs. They show that the relative deviation of the Cournot price from the Walrasian price is less or equal to the share of the largest company in the total production volume, divided by the demand elasticity. This estimate coincides with the Lerner index for the company and is precise if its marginal costs are equal for the Walrasian and the Cournot equilibria. Newbery (2009) considers these results in context of the data for Europian electricity markets. The share of the largest company typically exceeds 0.25, while the demand elasticity is less than 0.2. Thus, the data obviously contradicts to the Cournot model. Newbery calls it as Lerner Paradox and discusses different explanations considered below.

Let the demand elasticity at the Cournot outcome $e^* = dp^* / (\overline{D} - dp^*)$ meet condition $e^* > \alpha$. Then $\frac{p^*}{\widetilde{p}} - 1 \le \frac{1}{e^* / a - 1}$

This condition holds as the equality for a symmetric oligopoly with fixed c marginal cost $c = \tilde{p}$, and also for a large firm with fixed marginal cost interacting with the competitive environment characterized by smaller marginal cost and limited total capacity $V_F = (1-\alpha)D(p^*)$.



Why is Cournot outcome of any interest?

- 2) Vasin, Vasina, Ruleva (2007) consider a uniform price auction, where a strategy of each commodity producer is a non-decreasing step function that determines the actual supply of goods depending on the price. They show that, for any Nash equilibrium, the market price lies between the competitive equilibrium price and the Cournot price, and vice versa, each price in this range corresponds to a Nash equilibrium. However, only the Nash equilibrium corresponding to the Cournot outcome is stable with respect to the dynamics of adaptive strategies.
 - Note that Moreno & Ubeda (2002) obtain similar results for the two-step model where at the first step producers set capacities , and at the second step they compete by setting reserve prices.
 - Kreps & Sheinkman (1993) show that the SPE outcome of the two-stage model "first quantities, then prices" also corresponds to the Cournot equilibrium.



Proposition 1. Let the demand elasticity be a non-decreasing function. Then

a) For every Nash equilibrium without rationing, the production volumes correspond to the Cournot equilibrium. Vice versa, if $(v^a, a \in A)$ is the Cournot equilibrium, then the corresponding Nash equilibrium exists in Γ_s .

b) If $(R^a, a \in A)$ is a Nash equilibrium such that $D(\tilde{c}) \in (R^-(\tilde{c}), R^+(\tilde{c}))$, then there exists at most one producer $b \in A$ such that $R^{b-}(\tilde{c}) < S^{b-}(\tilde{c})$ (so $v^a \in S^a(\tilde{c})$ for any $a \neq b$); the cut-off price lies in the interval $[\tilde{p}, p^*]$.

c) For any Nash equilibrium of the type c), the cut-off price lies in the interval $[\tilde{p}, p^*]$. Vise versa, for any $p \in [\tilde{p}, p^*]$ there exists a Nash equilibrium $(R^a, a \in A)$ such that $\tilde{c}(R^a, a \in A) = p$.

3) Models by Baldick at al. (2000), Green (1992), Klemperer & Meyer (1989) describe the uniform price supply function auction with continuous bids as a game in normal form and characterize the Nash equilibria of the auction. Klemperer and Meyer study the competition model with arbitrary bid functions, including non-monotonic. For a given demand function they receive a lot of Nash equilibrium corresponding to all prices grater than the Walrasian price. Green and Newbery (1992) consider a symmetric duopoly with linear functions of supply and demand and get the formula for the calculation of the Nash equilibrium. Baldick et al. (2000) generalize the results to the asymmetric oligopoly. Abolmasov and Kolodin (2002) and Dyakova (2003) applied this approach to study electricity markets in two Russian regions. They use the affine approximations of the true supply functions and obtain a significant reduction of the "market power" in the supply function auction compared with the Cournot auction.

Can a model of SFE with linear supply functions and marginal costs adequately describe and explain the Lerner paradox? Note that the assumption of affine structure of the supply function does not correspond to the actual cost structure of energy companies, nor the practice of the auction. In a typical DAM every producer may submit a bid corresponding to a non-decreasing piece-wise step function. In a first approximation the real structure of the variable costs of many power companies also corresponds to such function. Usually, such a company owns several power generators with limited power, each of them is characterized by constant marginal costs. Their main component - the consumption of fuel and water. Under these conditions, the equilibrium bid is a nonlinear function of price. This is confirmed by the results obtained in the other direction of research initiated in the same paper, Klemperer & Meyer (1989).

An important feature of electricity markets is uncertainty of demand,

which is due to random changes of the environment and also to variations of the demand during the time for which the bids are submitted. In this context, Klemperer and Meyer (1989) proposed a promising auction model and theoretical results. They assumed a bid to be a monotone smooth function and the demand function to depend on a random parameter. Thus, the cut-off price that equalizes the total supply and demand is random. A bid profile is called supply function equilibrium (SFE) if for any parameter value the bid of each firm maximizes its profit under fixed bids of other producers. For a symmetric oligopoly, the authors derive a differential equation for an equilibrium bid and describe the set of the SFF.

The SFE price is always lower than the Cournot oligopoly price. In some cases, the price reduction is significant (Green, 1997, Newbery, 1998). On this ground, some researchers claim that the supply function auction is an efficient mechanism for reduction of the "market power" of producers.

- However, computation of the SFE bids is a rather sophisticated mathematical problem. In general, its solution requires full information on the demand function and the cost functions of all competitors. Why should one expect that the actual behavior at the auction corresponds to this concept?
 - A similar question for Nash equilibria of normal form games is considered in the framework of adaptive and learning mechanisms' investigation (see Milgrom, Roberts, 1990, Vasin, 2005). The study shows that for some classes of games rather simple mechanisms provide convergence of strategy profiles to stable NE for players with bounded rationality and incomplete information. But some NE are not stable in this sense.
- Vasin, Dolmatova (2010) and Vasin, Gusev (2011) consider best response dynamics in the SFA for two variants of a symmetric oligopoly with a linear demand function: A) with a linear marginal cost, B) with a fixed marginal cost and a limited production capacity. Our purpose was to find out for each case if the dynamics converges to any SFE.

A) The model with linear marginal cost function

• Consider a symmetric duopoly with cost function $C(q) = (c_0 + 0.5c_1q)q$, c1 > 0, where, $c_0 > 0$, $c_1 > 0$ and demand function D(p,t) = D(t) - dp, where d > 0 and D(t) is a maximal demand depending on random parameter t with a given distribution function. According to Klemperer and Meyer (1989), an equilibrium supply function for this case should meet differential equation

$$S'(p) = \frac{S(p)}{p - c_o - c_1 S(p)} - d$$

• If $\sup_{t} D(t) = \infty$ then there exists a unique SFE and the bid function is linear:

$$S^{*}(p) = 0,5(p-c_{0})d(-1+\sqrt{\frac{4}{dc_{1}}}+1)$$
 (2).

• Consider best response dynamics for the repeated auction in this case. At every step each firm sets bid $S(p,\tau)$ that is the best response to its competitor's bid $S(p,\tau-1)$ at the previous step. (We assume S(p,0)=0).

Formally, $S(p,\tau)$ is the best response to $S(p,\tau-1)$ if $\forall t \quad p(\tau,t)$ that is a solution to $S(p,\tau)+S(p,\tau-1)=D(p,t)$, also provides the maximal profit: $p(\tau,t) \rightarrow \max_{p} [(D(p,t)-S(p,\tau-1))p-C(D(p,t)-S(p,\tau-1))]$

Though the demand depends on random parameter t, at every step there exists a bid that maximizes the profit under any value of this parameter.

•**Proposition 3.** Bid $S^{1}(p) = \frac{(p-c_{0})(d+k)}{1+c_{1}(d+k)}$ is the best response to bid $S^{2}(p) = k(p-c_{0})$ under any $D(t) > dc_{0}$. Thus, the best response bid at step τ is $S(p,\tau) = k_{\tau}(p-c_{0})$ where $k_{\tau} = \frac{d+k_{\tau-1}}{1+c_{1}(d+k_{\tau-1})}$ The unique fixed point $k^{*} = \frac{d}{2}(\sqrt{\frac{4}{dc_{1}}+1}-1)$ for this equation corresponds to the SFE (2) of the auction. (See also Rudkevich, 1999)

Proposition 4. Best response dynamics for model A converges to the SFE (2). Moreover, $|\frac{k_{\tau}}{k^*}-1| \leq |\frac{k_1}{k^*}-1| (1+c_1d)^{\tau-1}$.

B) The model with fixed marginal cost and capacity constraint

- In this case C(q) = cq, c > 0, and $q \le Q$. SFE bid for this case is a continuous monotone function that meets equation $S'(p) = \frac{S(p)}{(p-c)} d$ until S(p) = Q for some p.
- A general solution $S(p,A) = (p-c)(A d \ln(p-c))$ for this equation depends on integration constant A. It reaches max value q(A) under $p(A) = c + \exp(A/d - 1)$ $A(q) = d(\ln(q) - \ln(d) + 1)$
- Denote $D^* = \sup_t \overline{D}(t) dc$. We define function $D^*(Q)$ and inverse function $q(D^*)$ proceeding from the system: $D^* d(p-c) = 2Q = 2d(p-c)$ In particular, for d = 1, c = 0 $D^*(Q) = 3Q$, $q(D^*) = D^*/3$

SFE depending on the maximum demand $S^{*}(p) = \begin{cases} S(p, A(Q)), p < c + Q/d \\ Q, p \ge c + Q/d \end{cases}$ $-2S^{*}(p)$ $-2\overline{S}(p)$ $-2\overline{\overline{S}}(p)$ (1) 1 views $\overline{S}(p,A) = \begin{cases} S(p,A), p < p(A) \\ q(A), p \ge p(A) \end{cases}$ ----2d(p-c) 3Q+dc 2 views $\overline{\overline{S}}(p,A) = \min\{\overline{S}(p,A), Q\}$ **Proposition 5.** 1) If $D^* \ge 3Q$, then \exists SFE (2) $S^*(p) = \min\{S(p, A(Q)), Q\}$ 2Q2) If $Q < D^* < 3Q$, then $\forall A \in (A(D^*/3), A(Q))$ Q+dc $\overline{S}(p,A)$ - SFE of type 1 $\forall A \in (A(Q), \overline{A}(D^*))$ (3) $\overline{S}(p,A)$ - SFE of type 2 If $D \leq Q$, then $\forall A \in A(D^*/3)$ 3) $\overline{S}(p,A)$ - SFE of type 1

Now consider best response dynamics.

• Proposition 6. Best response dynamics for $D^* \leq Q$ is $S(p,\tau) = \begin{cases} \tau d(p-c) & \text{for } p < \frac{Q}{\tau d} + c \\ Q & \text{for } p \ge \frac{Q}{\tau d} + c \end{cases}$

It converges to the SFE that corresponds to Walrasian equilibrium for this case. Consider $D^* \ge 3Q$. Let c = 0, d = 1.



Thus, there is no monotonous BR function for the third step, and the BR dynamics does not converge to the SFE.

- Consider another approach. We fix the value of \overline{D} and define the BRD depending on the ratio between \overline{D} and Q.
- <u>Proposition 7.</u> The best response dynamics depends on the parameter values as follows:
- For $\overline{D} \ge 3Q$ $S(p,\tau) = \min\{Q,dp\}, \forall \tau$. For any τ the SFE coincides with the Cournot supply schedule. The outcome corresponds to the Walrasian equilibrium that is equal to the Cournot equilibrium in this case.
- For $Q < \overline{D} < 3Q$ $S(p, \tau) = \min\{Q, d\tau p\}$, at step $\tau = 1, ..., T(\overline{D})$, then the BR functions repeat in cycle. The length of the best response dynamics cycle $\sqrt{2 - \frac{7}{2}} = \sqrt{2}$

$$T(\overline{D}) = \begin{cases} 2, & \text{for} \quad \frac{7}{3}Q < \overline{D} < 3Q \\ \\ 3, & \text{for} \quad 2 < \overline{D} \le \frac{7}{3}Q \\ \\ \left|\frac{\overline{D}}{\overline{D} - Q}\right|^2, & \text{for} \quad Q < \overline{D} \le 2Q \end{cases}$$

For $\overline{D} \le Q$ $S(p,\tau) = \min\{Q, d\tau p\}, \forall \tau$. For $\tau \to \infty$ the BRD converges to the SFE with the outcome corresponding to the Walrasian equilibrium.

There is no convincing empirical data showing that real markets perform according to the SFE model. Newbery (2009) discusses the statistics on different markets, and notes that the observed decline in the "market power" allows alternative explanations. The most significant is, in our view, the assumption of the role of the forward market, which exists in all of referred electricity markets.

5) An alternative possibility of SF auction organization, considered in several studies (Ausubel, Cramton, 1999, Bolle, 2004 Vasin, Vasina, Ruleva, 2007), is to use the Vickrey auction. At this auction the cut-off price and production volumes are determined in the same way as in the uniform price auction. However, each producer is paid her reservation price for her goods. The marginal price is the minimum of the marginal cost of the same output for other producers and the marginal reservation price of this output for consumers. The marginal cost is calculated on the basis of the reported supply functions, but in this case reporting actual costs and production capacities is a weakly dominant strategy. In the absence of information on production costs, the guaranteed total welfare reaches its maximum at the corresponding Nash equilibrium, and each producer makes a profit equal to the increment of the total welfare of all participants in the auction as a result of his participation in the auction.

Our calculations for the Central Economic Region of Russia show that the Vickrey auction price for consumers exceeds the Walrasian price at 50% (to compare with 250-400% for the Cournot price). However, such increase seems to be also rather essential. Besides, there exists a reasonable arguments implying that the participants of Vickrey auction typically do not reveal their actual costs, that is, the specified equilibrium in dominant strategies is not realized (see Rothkopf et al., 1990). The main argument is that reporting actual costs gives an advantage to the

auctioneer (and also to other economic partners) in further interactions with this producer.

• The situation is different if marginal costs and maximal capacity of each generator are common knowledge, and uncertainty pertains to a decrease of the capacities due to breakdowns and repairs. In this case the current state of working capacities is weakly correlated with the future state, and the specified argument against revealing the actual costs loses its validity. Moreover, available information may be used for redistribution of the total income in favor of consumers.

Vickrey auction with reserve prices

The set of strategies $\{R^{a}(p), p \ge 0\}$ of each participant, the rule for the cut-off price $\tilde{c}(R^{a}(p), a \in A)$ and production volumes are the same as discussed for the single price auction. Producer *a* payment is calculated as follows. Reserve price for the additional volume of the goods dv release of v^{a} is participant *a* - $\min\{(R^{A\setminus a})^{-1}(R^{A\setminus a}(\tilde{c})+v^{a}), D^{-1}(R^{A\setminus a}(\tilde{c})+v^{a})\}$.

The first function specifies a price limit for this amount, which would have to pay if you exclude participant *a* from the auction. The price is determined on the basis of the stated supply functions of other players: $R^{A \setminus a}(p) = \sum_{b \in A \setminus a} R^{a}(p)$. The second function defines a reserve price that consumers are willing to pay for this volume. Profit player *a* is

$$f^{a}(R^{a}, a \in A) = \int_{0}^{R} \min\{(R^{A \setminus a})^{-1}(R^{A \setminus a}(\widetilde{c}) + v^{a}), D^{-1}(R^{A \setminus a}(\widetilde{c}) + v^{a})\}dv - C^{a}(R^{a}(\widetilde{c}))$$

Power generator	Marginal cost (rubles/MVt)	Maximum production (GVt × year)	<u>GC1:</u>	1 2	0 60	16 2
				3	112	3
				4	125	2
G1	0	5		5	150	16
01	0	5		6	200	2
G2	75	10		7	255	2
				8	340	10
G3	80	10	<u>GC2:</u>			
G4	85	25		1	95	2.5
05	0.0	10		2	110	2.5
GS	90	10		3	120	4
G6	100	5		4	128	13
~	1.65	10		5	135	6
G7	165	10		6	145	2
				7	162	15
			<u>GC3:</u>			
				1	0	3.5
	12.5	125 /		2	100	2.5
	12.5	123.4		3	120	21
				4	150	3.5

4.5 4.5

The linear demand function $D(p) = N - \gamma p$, corresponds to the data on consumption in 2000:

Two variants of the market structure:

a) consisting of 5 companies

b) consisting of 3 (Mosenergo, Rosenergoatom and UGC that includes all the other generators).

Table 1. Walrasian price and the ratio for the Cournot and Vickrey prices tothe Walrasian price in the Central economic region of Russia

γ p γ	\widetilde{p}	p_5^*/\widetilde{p}	p_3^*/\widetilde{p}	p_{V5}/\widetilde{p}	p_{V3}/\widetilde{p}	$\overline{p}_{V5}/\widetilde{p}$	$\overline{p}_{V3}/\widetilde{p}$
0,1	135	4.24	5.65	1.59	2.19	0.51	0.62
0,2	150	2.45	3.10	1.49	1.92	0.44	0.57
0,4	172.5	1.56	1.87	1.49	1.76	0.42	0.49
0,6	219.67	1.15	1.34	1.30	1.46	0.33	0.38

- Another possible form of the auction is a "pay-as-bid" auction. Sales
 volumes are defined in the same way as for a uniform price auction, but
 the payment is made to each participant according to the prices specified
 in her bid. This form was used for the electricity market in England and
 Wales, as well as in Russia in the capacity market.
- As a trivial argument in its favor, we note that, for fixed bids, the sales price • for consumers is reduced compared to the uniform price auction. However, this form has serious drawbacks. Rational behavior of participants is significantly different from the above options. Even under conditions of perfect competition, submission of a bid corresponding to real costs is unreasonable. The optimal strategy for a producer is to calculate the competitive equilibrium price and to offer at this price the corresponding amount. Given the incompleteness of the information, it is practically impossible. In the case of imperfect competition, the Nash equilibrium in the corresponding game typically does not exist, because the auction is similar to the Bertrand-Edgeworth model of price competition. This situation is pushing sellers to conclude cartel agreements as a means to ensure the stable operation of the market. This, of course, increases their bargaining power. Therefore, in our opinion, everyone should agree with K. Wolfram, who does not recommend this type of an auction.

The final part of our survey is devoted to the role of the forward market in reducing the market power of large companies.

James Bushnell (2005) considered a two-stage Cournot auction with a constant marginal cost, and showed that the ability to make forward contracts reduces the bargaining power of producers as well as an increase in their number in the market from n to n^2 .

Note the following problems related to the latter study. First, the actual price trends in the electricity markets are not consistent with the hypothesis of equality of prices in the spot and forward markets. Usually the price in the spot market is slightly lower, but sometimes there are jumps in which the spot price significantly exceeds the price in the forward market. The second problem relates to the assumption of the priority of consumers with high reserve prices when buying goods in the forward market. It is hard to imagine the possibility of such a distribution of consumers without special rationing, which does not exist in real markets.

Vasin et al (2009) and Vasin, Daylova (2012) consider a two-stage model with a random market price in the spot market. We take into account the presence of risk-neutral arbitrageurs, the competition between them leads to equality of the forward price to the spot price expectation. Consumers operate under conditions of perfect competition and are free to choose between the spot and forward markets. Our model describes a strategic interaction between producers, consumers and arbitrageurs. We find the optimal strategies of rational consumers, depending on the reserve price and the parameter characterizing risk aversion. We examine properties of the subgame perfect equilibrium (DSS) for the model under the assumption that the proportion of risk-preferring consumers with high reserve prices is constant.

In our model at the equilibrium the producers employ correlated mixed strategies, and the corresponding outcome is random: the expected (rather than actual) spot market price coincides with the price in the forward market. Consumers with low reserve prices buy goods at the spot market if the price is lower than their reserve price, otherwise they refuse the purchase. The risk-preferring consumers with high reserve prices always buy goods at the spot market. Risk-averse consumers buy in the forward market if their reserve price is higher than the forward price and the risk aversion parameter is above a certain threshold.

Fluctuations of the spot price are usually explained by the existence of random external factors. Our model shows that external factors are not necessarily the main reason. In the game describing the spot market there are two local equilibria. The first (with the low price) corresponds to the steep slope of the residual demand $(p < p^{f}, "bear market")$. The second (with the high price) corresponds to the small slope of the residual demand ($p > p^{f}$, "bull market"). In the subgame perfect equilibrium in the spot auction "bear market" with lower prices realizes often, "bull market" with higher prices - seldom.

The equilibrium distribution of consumers between the forward and the spot market



Examples: A fixed share of risk preferring consumers, two possible prices at the spot market

Case 1: For consumers with $\lambda > 0$ and $p^f < r_b < p_2$ risk aversion is so high that all of them buy at the forward market .

Case 2: For consumers with $\lambda > 0$ and $p^f < r_b < p_2$ risk aversion is so low that all of them ignore the forward trade and wait for the spot sales .



The influence of the forward market on the bargaining power of producers

$p^{f}-c$	α = 0. I		<i>α</i> = 0.3		a = 0.7		<i>α</i> = 0.9		Bushnel's
Δ^*	<i>w</i> ₁	w_2	w_1	<i>w</i> ₂	w_1	<i>w</i> ₂	w_1	<i>w</i> ₂	result
n = 2	-	-	0.7724	0.8292	0.6531	0.6324	-	-	0.6
n = 3	0.7551	0.6617	0.5953	0.4922	0.4578	0.4284	0.4169	0.4084	0.4
n = 4	0.6509	0.4252	0.4808	0.3689	0.3468	0.3171	0.3093	0.3009	0.29
n = 5	0.5728	0.3390	0.4021	0.2923	0.2773	0.2496	0.2441	0.2364	0.23
n = 6	0.5117	0.2805	0.3450	0.2410	0.2304	0.2051	0.2009	0.1939	0.18
n = 7	0.4625	0.2387	0.3018	0.2045	0.1967	0.2101	0.1703	0.1640	0.16
n = 8	0.4220	0.2074	0.2681	0.1774	0.1714	0.1504	0.1477	0.1420	0.13
n = 9	0.3880	0.1832	0.2411	0.1565	0.1518	0.1325	0.1303	0.1251	0.12
n = 10	0.3591	0.1639	0.2190	0.1400	0.1362	0.1184	0.1165	0.1117	0.1

What is the situation with short-term performance of the Russian electricity market? DAM is organized as a uniform price auction, rather, as an auction with uniform nodal prices (the network structure is important for the Russian market). Market analysis shows that in some regions the potential market power of large companies is high. However, in practice, there is no large deviations from the Walrasian market prices arising from the estimates and calculations for the Cournot auction. However, neither the supply function auction mechanism nor the market of forward contracts produce this effect. In reality the market prices are limited by the state regulatory agencies that are interested in maintaining a stable and low prices for households and large enterprises. The back side of this regulation is a very high cost of connecting new capacities to consumers. With the reduction in the use of "manual control" in the market, the issues discussed above will become relevant to its development.

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