On Incomplete Fuzzy and Multiplicative Preference Relations In Multi-Person Decision Making

ITQM 2014

R. Ureña[1], F. Chiclana[2], S. Alonso[3], J.A. Morente-Molinera[1], E. Herrera-Viedma[1]

[1]Dept. Computer Science and Artificial Intelligence, University of Granada, Granada, Spain

[2]Centre for Computational Intelligence, Faculty of Technology, De Montfort University, Leicester, UK

[3]Dept. Software engineering, University of Granada, Granada Spain

4 June 2014

Contents

Motivation

- Rapid changes in business environment.
- Geographical dispersion of firms.
- Globalization
- Complex decision involving many alternatives

$\Downarrow$

Group decision support systems

Goal

How should we deal with incomplete information in Group decision making?

Group decision making

- Group decision making (GDM) consist of multiple individual interacting to choose the best option between all the available ones.
- Experts have to express their preferences over a set of alternatives (Pairwise).

Definition

A preference relation P on the set X is characterized by a function $\mu_p : X \times X \rightarrow D$, where D is the domain of representation of preference degrees provided by the decision maker for each pair of alternatives.

Preference relations

Definition (Additive Preference Relation (APR))

An APR P on a finite set of alternatives X is characterised by a membership function $\mu_P \colon X \times X \longrightarrow [0,1],\ \mu_P(x_i, x_j) = p_{ij}$, verifying $p_{ij} + p_{ji} = 1\ \forall i, j \in \{1, \ldots, n\}$.

Definition (Multiplicative Preference Relation (MPR))

A MPR A on a finite set of alternatives X is characterised by a membership function $\mu_A \colon X \times X \longrightarrow [1/9, 9],\ \mu_A(x_i, x_j) = a_{ij}$, verifying $a_{ij} \cdot a_{ji} = 1\ \forall i, j \in \{1, \ldots, n\}$.

Proposition

Suppose that we have a set of alternatives, $X = \{x_1, \ldots, x_n\}$, and associated with it a MPR $A = (a_{ij})$, with $a_{ij} \in [1/9, 9]$ and $a_{ij} \cdot a_{ji} = 1, \forall i, j$. Then the corresponding APR, $P = (p_{ij})$, associated to A, with $p_{ij} \in [0,1]$ and $p_{ij} + p_{ji} = 1, \forall i, j$, is given as follows:

$$p_{ij} = f(a_{ij}) = \frac{1}{2}\left(1 + \log_9 a_{ij}\right) \quad (1)$$

Consistency of Preference relation

There are three fundamental and hierarchical levels of rationality assumptions when dealing with preference relations

1. Indifference between any alternative x_i and itself.
2. If an expert prefers x_i to x_j, that expert should not simultaneously prefer x_j to x_i.
3. Transitivity in the pairwise comparison among any three alternatives. if x_i is preferred to x_j ($x_i \succ x_j$) and this one to x_k ($x_j \succ x_k$) then alternative x_i should be preferred to x_k ($x_i \succ x_k$), which is normally referred to as *weak stochastic transitivity*

Consistency of APR and MPR

Definition (Consistent MPR)

A MPR $A = (a_{ij})$ is consistent if and only if

$$a_{ij} \cdot a_{jk} = a_{ik} \; \forall i, j, k = 1, \ldots, n.$$

Definition (Additive consistency of APR)

An APR $P = (p_{ij})$ on a finite set of alternatives X, it is additive consistent if and only if

$$(p_{ij} - 0{,}5) + (p_{jk} - 0{,}5) = p_{ik} - 0{,}5 \;\; \forall i, j, k = 1, 2, \cdots, n$$

Definition (Multiplicative consistency of APR)

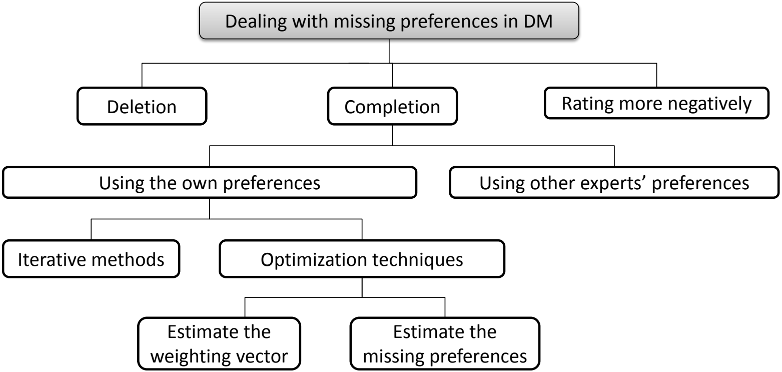
An APR $P = (p_{ij})$ on a finite set of alternatives X is multiplicative consistent if and only if

$$p_{ij} \cdot p_{jk} \cdot p_{ki} = p_{ik} \cdot p_{kj} \cdot p_{ji} \;\; \forall i, k, j \in \{1, 2, \ldots n\}$$

Incomplete information

- Expert might not possess a precise or sufficient level of knowledge of part of the problem
  - high number of alternatives
  - limited time,
  - not enough knowledge of a part of the problem
  - conflict in a comparison

General view of the estimation approaches



Iterative approaches

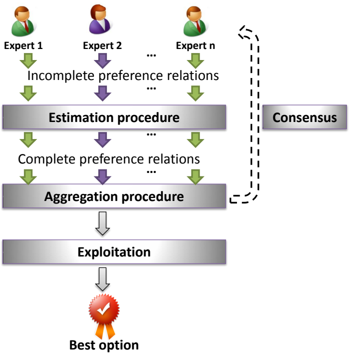
- Uses intermediate alternatives to create indirect chains of known preference values, (p_{ik}, p_{kj}), to derive p_{ij} $(i \neq j)$, using the additive consistency property. [1]

$$ep_{ij}^{k} = p_{ik} + p_{kj} - 0{,}5.$$

The *overall consistency based estimated value* is obtained:

$$ep_{ij} = \sum_{k=1, k \neq i,j}^{n} \frac{ep_{ij}^{k}}{n-2}$$

- Extension to work with IVPR, LPR, MPR.



[1] Herrera-Viedma, E., Chiclana, F., F.Herrera, Alonso, S., 2007. Group decision-making model with incomplete fuzzy preference relations based on additive consistency. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics* 37 (1), 176–189

Optimisation based completion approaches

Maximizes the consistency and/or the consensus of the experts' preferences. [2]

- Minimizes the *global additive inconsistency* index of the incomplete APR

$$\rho = 6 \cdot \sum_{i<k<j} L_{ijk}$$

  where

$$L_{ijk} = (p_{ik} + p_{kj} - p_{ij} - 0{,}5)^2$$

- Maximize the consistency level proposed by Herrera-Viedma et al. To increase the individual consistency defines a linear optimisation method that minimises the Manhattan distance between the provided preference relation and the completed consistent based one[3]

[2] Fedrizzi M., Giove S. Incomplete pairwise comparison and consistency optimization. European Journal of Operational Research 2007. 183(1):303–13.

[3] Zhang G., Dong Y., Xu Y. Linear optimization modeling of consistency issues in group decision making based on fuzzy preference relations. Expert Systems with Applications 2012;39:2415?20.

Priority weights computation

They directly rank the alternatives without completion:

Based on Saaty's assumption for MPR regarding the exact functional relation between the preference values and the priority vector.

- Linear system of equation in which the missing values are substituted by their relation with the priority vector.
- Goal programming models
- Least square minimization

Which one is better?

- Comparative study of seven different methods for reconstructing incomplete APR and MPR in terms of the Saaty's consistency Ratio[4].
- 4 methods for MPR and three for APR using both, consistent and highly inconsistent preference relations.
- Results:
  1. Optimization methods where missing entries are directly computed.
  2. Methods where priority weights w_i are first computed.
  3. Least square approaches.

[4] Brunelli M., Fedrizzi M., Giove S., Reconstruction methods for incomplete fuzzy preference relations: A numerical comparison. In: Proceedings of the 7th international workshop on Fuzzy Logic and Applications: Applications of Fuzzy Sets Theory. Berlin, HeidelbergWILF. Springer-Verlag, Berlin, Heidelberg: WILF ?07. Springer-Verlag; 2011, p. 86?-93

Processes dealing with ignorance situations in GDM

Previous approaches are not useful when information about a particular alternative is not known. [5]

- Individual strategies: Seed value
  - indifference
  - proximity
- Social strategies: Consensus preference values from all the experts and from the nearest ones.

[5] Alonso S., Herrera-Viedma E., Chiclana F., Herrera F. Individual and social strategies to deal with ignorance situations in multi-person decision making. International Journal of Information Technology and Decision Making 2009;8(2):313–33

Conclusions

- We have reviewed the main completion approaches in the literature to deal with missing information for APR and MPR including total ignorance situations.
- The majority of the approaches uses the additive or the multiplicative properties to estimate the missing values from the known ones.
- They can be broadly classified as iterative approaches and optimization approaches.
  - iterative approaches
  - optimisation approaches: maximize the consistency and/or the consensus of the experts' preferences.
    - goal programming
    - least square minimization

Questions?

Thanks for your attention