Time Series Forecasting Using Nonlinear Dynamic Methods and Identification of Deterministic Chaos

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Fundamentals of nonlinear dynamic

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Exceptional feature *of nonlinear systems is the ability to implement different developmental variations which are dependent on the initial state of the systems, system parameters and random disturbances.*

What is deterministic chaos?

Implementation of irregular oscillation regime similar to a random process but strictly predictable in the sense of the law of evolution.

Actuality

Complexity of solving the problem of data processing which contain chaotic component is:

- *Short time series*
- *The implementation process is unique*
- *No information about the probability distribution of errors.*

Objective

To improve the forecast accuracy by the identification of the chaotic component of the time process using deterministic nonlinear dynamic systems with chaotic solutions.

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Application

Engineering, cryptography, medicine, economics and other fields of science and technology.

The idea

To approximate residuals $e_k = y_k - y_k, k = 0,1,\ldots, N-1$ *by a linear combination of logistic map*

$$e_k = \sum_{i=1}^{m} a_i x_{ik} + \varepsilon_k, x_{ik+1} = \lambda_i x_{ik}\left(1 - x_{ik}\right), k = 0,1,\ldots, N-1.$$

Problem statement

The model of chaotic process

$$y_k = \sum_{i=1}^{n} a_i x_k^{(i)} + \eta_k,\ k = 1,2,\ldots, N$$

$$x_{k+1}^{(i)} = f_i\left(x_k^{(i)}, \lambda_i\right),\ k = 0,1,\ldots, N-1,\ i = 1,2,\ldots, n,$$

$$x_{k+1}^{(i)} = \lambda_i x_k^{(i)}\left(1 - x_k^{(i)}\right),\ k = 0,1,\ldots, N,\ i = 1,2,\ldots, n$$

$$x_0^{(i)} \in (0,1),\ \lambda_i \in (\lambda_\infty, 4],\ \lambda_\infty \approx 3.57$$

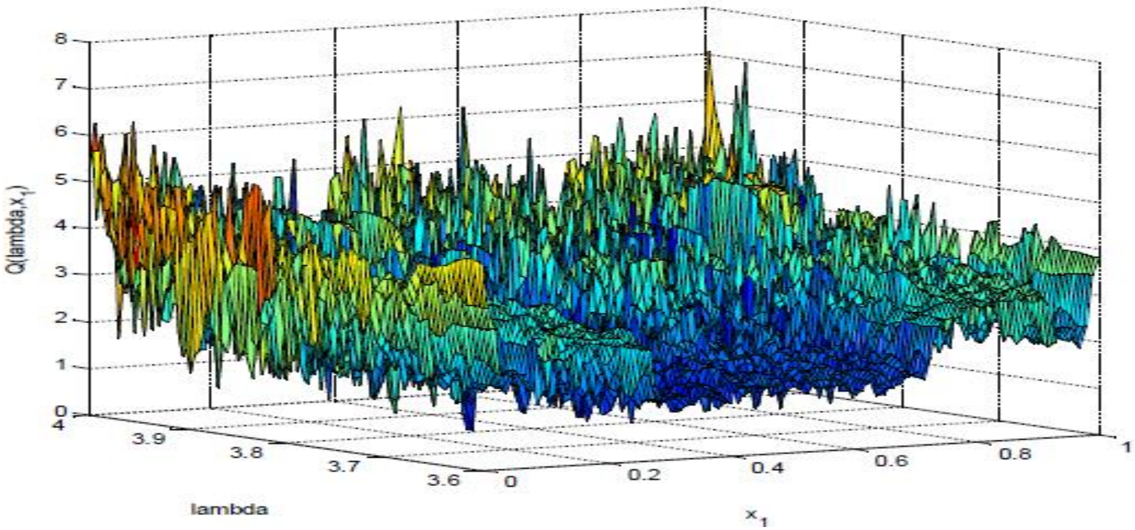

Fig.1. The objective function

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Data preprocessing

Frequency analysis and selection of periodic components

1. Discrete Fourier transform (DFT) $Y_t = \sum_{k=1}^{N} y_k e^{-\frac{2\pi i t}{N}(k-1)}$, $Y_t \in C$, $t = 0, 1, \ldots, N$.

The coefficients of the Fourier series $a_t = Y_t + \overline{Y_t} = \mathrm{Re}\, Y_t$, $b_t = -Y_t + \overline{Y_t} = \mathrm{Im}\, Y_t$, $t = 0, 1, \ldots, \frac{N}{2}$.

2. The model of deterministic component

$$\left.\begin{array}{l} x_{k+1} = f(\alpha, x_k) + \xi_k,\ k = 0,1,\ldots,N-1; \\ y_k = G x_k + \eta_k,\ k = 1,2,\ldots,N, \end{array}\right\},$$

$$\left.\begin{array}{l} x_{1k+1} = x_{1k} + x_{2k} + \xi_{1k}; \\ x_{2k+1} = x_{2k} + x_{3k} + \xi_{2k}; \\ \ldots \\ x_{nk+1} = x_{nk} + f(\alpha, x_{1k}, x_{2k}, \ldots, x_{nk}) + \xi_{nk},\ k = 0,1,\ldots,N-1; \\ y_k = x_{1k} + \eta_k,\ k = 1,2,\ldots,N. \end{array}\right\}.$$

$x_k \in \mathrm{R}^n$ – state vector of a dynamic systems, which law of development is described by $f(\,;\,)$, $y_k \in \mathrm{R}$ – measurement vector, $\xi_k \in \mathrm{R}^n$ – noise vector of the system, $\eta_k \in \mathrm{R}^m$ – interference vector of the measurements, $\alpha \in \mathrm{R}^p$ – vector of parameters, N – series length.
Estimation of the model parameters can be found using the Kalman filter.

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The reconstruction of the model of deterministic chaos

Deterministic chaos *is Implementation of irregular oscillation regime similar to a random process but strictly predictable in the sense of the law of evolution.*

The consequences the exponential instability:

- *nonperiodicity depending on time of any of the coordinates of the phase vector;*
- *continuous power spectrum;*
- *decreasing autocorrelation function;*
- *Hurst exponent (antipersistent ($0 \le H < 0.5$), random ($H \to 0.5$), persistent ($0.5 < H \le 1$);*
- *Lyapunov characteristic exponent > 0.*

Chaotic component identification of short time series

$x_{k+1}^{(i)} = \lambda_i x_k^{(i)}\left(1 - x_k^{(i)}\right),\ k = 0,1,\ldots,N,\ i = 1,2,\ldots,n$ *with chaotic solutions* $x_0^{(i)} \in (0,1),\ \lambda_i \in (\lambda_\infty, 4],\ \lambda_\infty \approx 3.57.$

In order to check the UKF it is considered a model example when the number of basic processes is 1 and the absence of noise in the system:

$y_k = a x_k + \eta_k,$

$x_k = \lambda x_{k-1}\left(1 - x_{k-1}\right),\ k = 1,2,\ldots,N,$

$x_k \in R$ *– chaotic solution of the system,* $y_k \in R$ *– measurement vector,* λ *– logistic map parameter,* $\eta_k \sim N(0,\sigma)$ *– noise measurement.*

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Practical results

Logistic map

$x_0 = 0.3,\ \lambda = 3.69,\ SNR = 10dB,\ REE = 10\%,\ RPE\ (8steps) = 10\%$

Parameter estimation error $\lambda = 3{,}65$ does not exceed 0,04 (the number of measurements N = 60)

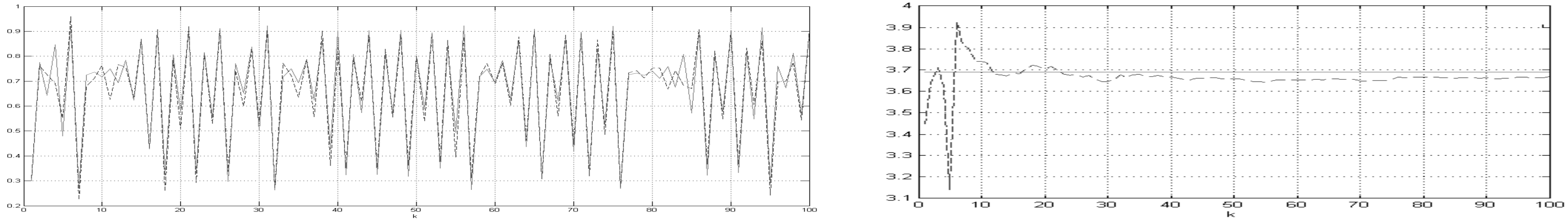

Fig. 2. (a) The investigated process x_k (–) and approximation (- -), (b) The estimation (- -), true value (–) of the parameter λ

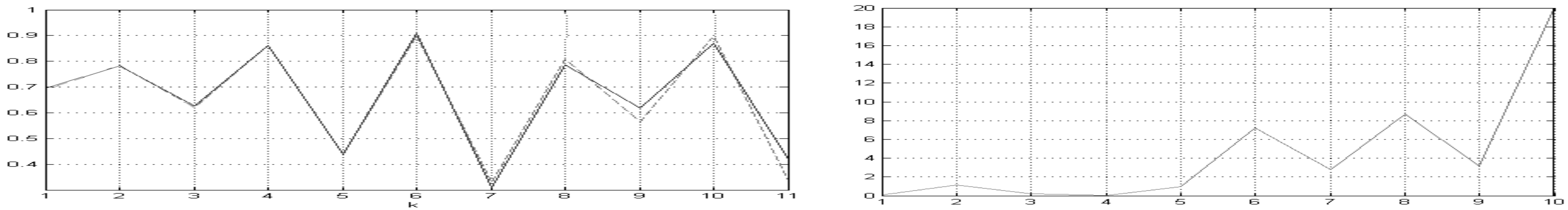

Fig. 3. (a) The forecast of the initial process, (b) Relative prediction error

The reason of the divergence of the forecast and the original series is in nature of logistic map with chaotic solutions when small deviations of the parameters λ, x_0 lead to the divergence of the processes.

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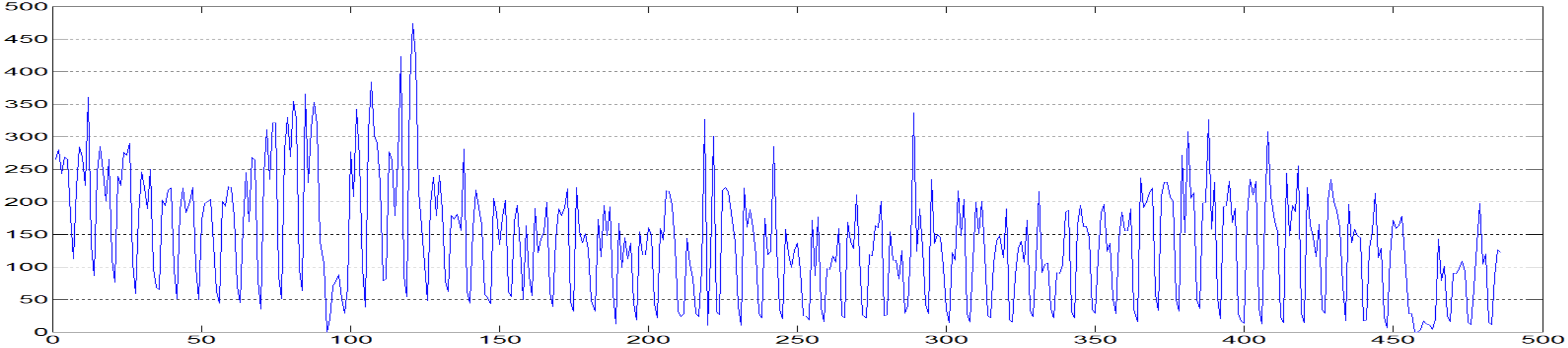

Fig.4. The communication traffic ($H = 0,7393$)

Fig.5. Frequency power spectrum of the initial time series

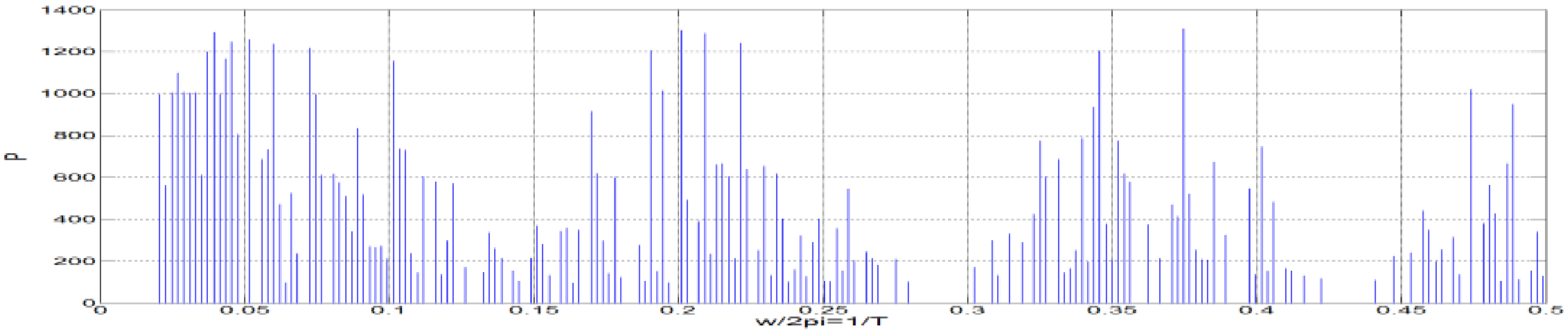

Fig.6. Frequency power spectrum of the time series after selecting weekly component

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Trend forecast

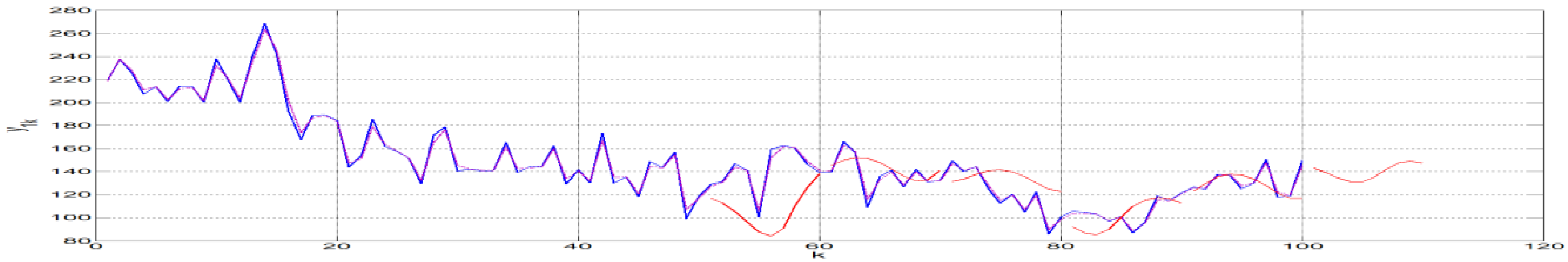

Fig.7. The forecast (RPE=7.92%)

The model of the residues e_k is following:

$$e_k = \sum_{i=1}^{m} a_i x_{ik} + \varepsilon_k,\ x_{ik+1} = \lambda_i x_{ik}\left(1 - x_{ik}\right),\ k = 0,1,\ldots,N-1,\ m = 3.$$

1 step: $\lambda_1 = 3{,}6,\ \lambda_2 = 3{,}7,\ \lambda_3 = 3{,}8,\ x_{i0} = 0{,}5,\ i = 1,2,3.$ H=0.4492.

2 step: $\lambda_1 = 3{,}65,\ \lambda_2 = 3{,}72,\ \lambda_3 = 3{,}78,\ H = 0{,}4820$

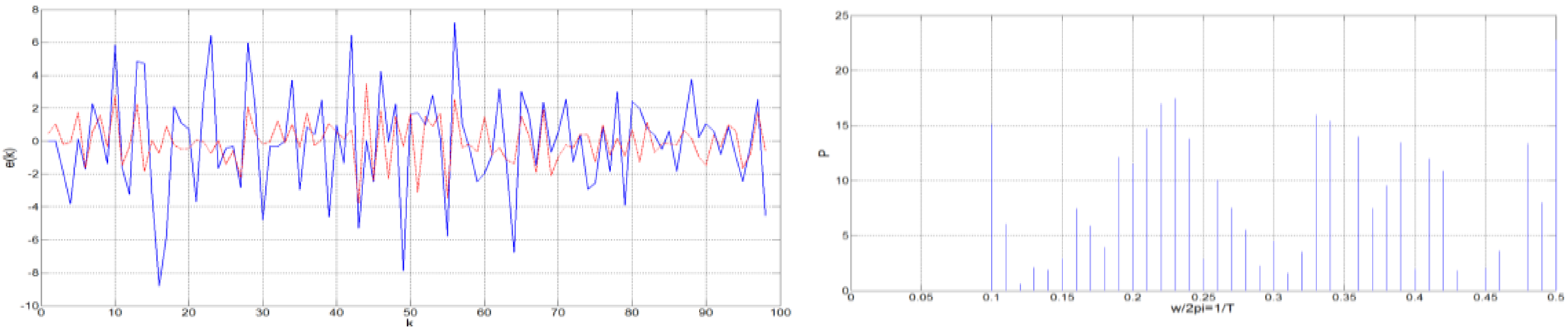

Fig.8. (a) The residues e_k (H=0.4219) and approximation of residues, (b) Frequency power spectrum of the residues

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Conclusion

The study proposes an approach to the modeling of the process model using the methods of nonlinear dynamics, as well as to the modeling of a chaotic process in terms of a small number of available measurements and the only implementation of the process.

There are the criteria of deterministic chaos: nonperiodicity depending on time of any of the coordinates of the phase vector, the power spectrum is concentrated in the low frequency band, decreasing autocorrelation function, Hurst exponent, Lyapunov characteristic exponent.

The parameters of the model process as logistic map were estimated using UKF. Convergence of parameter estimation was obtained at a short interval sampling $N = 60$. The absolute error of parameter estimation does not exceed 0.04. Relative forecast error up to 8 steps does not exceed 10%.

The model of the communication traffic of the telecommunication market was built in the study. The periodic components were isolated using DFT. In order to increase the forecast accuracy the resulting residues were approximated by the sum of logistic maps because of the presence of indications of deterministic chaos.

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