On the superposition of the Borda and threshold preference orders for three-graded rankings

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Introduction

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Given a finite set of *alternatives* X and a *preference order* P on X, we would like to scale X to real numbers by means of a function, preserving order properties. More precisely, $F : X \to \mathbb{R}$ is a *utility function* for P if, given $x, y \in X$,

$$(x, y) \in P \quad (x \text{ is } P\text{-preferred to } y) \quad \text{iff} \quad F(x) > F(y).$$

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Thus, $X = \{1, 2, 3\}^n$, and $x \in X$ means that $x = (x_1, \ldots, x_n)$ with coordinates $x_i \in \{1, 2, 3\}$ ($n \geq 3$).

Three preference orders will be considered on X:

1) the Borda preference order,
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Outline

1. Preference orders
   - Borda and threshold preference orders
   - Superposition of preference orders

2. Results
   - Axiomatics of utility functions for $B * V$
   - The enumerating utility function

Outline

We begin by recalling a few well-known definitions.

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(Preference orders are also called *weak orders*.)

Notation: $x \succ_P y$ denotes $(x, y) \in P$ (x is P-preferred to y).

The *indifference relation* I_P on X is defined as the set of all pairs $(x, y) \in X \times X$ such that $x \not\succ_P y$ and $y \not\succ_P x$.

$x \approx_P y$ denotes $(x, y) \in I_P$ (x and y are P-indifferent).

Example. If $F : X \to \mathbb{R}$ is a nonconstant function, then the set $P(F)$ of all pairs $(x, y) \in X \times X$ such that $F(x) > F(y)$ is a preference order on X. We have: $x \approx_{P(F)} y$ iff $F(x) = F(y)$.

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Notation: $x \succ_P y$ denotes $(x, y) \in P$ (x is P-preferred to y).

The *indifference relation* I_P on X is defined as the set of all pairs $(x, y) \in X \times X$ such that $x \not\succ_P y$ and $y \not\succ_P x$.

$x \approx_P y$ denotes $(x, y) \in I_P$ (x and y are P-indifferent).

Example. If $F : X \to \mathbb{R}$ is a nonconstant function, then the set $P(F)$ of all pairs $(x, y) \in X \times X$ such that $F(x) > F(y)$ is a preference order on X. We have: $x \approx_{P(F)} y$ iff $F(x) = F(y)$.

The preference order $P(F)$ is called *F-representable*.

Borda preference order

Set $S(x) = x_1 + \cdots + x_n$ if $x = (x_1, \dots, x_n) \in X = \{1, 2, 3\}^n$.
Given $x, y \in X$, $x \succ_B y$ (*x is Borda preferred to y*) if $S(x) > S(y)$.
B is a preference order on X with 'coarse' ranking of X.

Example. Let $n = 5$ and $x = (x_1, \dots, x_5)_N$ be a representative of the indifference class with $x_1 \leq \cdots \leq x_5$ and $N = S(x) - 4$ (ordering in ascending B-preference):

$(1,1,1,1,1)_1$, $(1,1,1,1,2)_2$, $(1,1,1,1,3)_3$, $(1,1,1,2,2)_3$,
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Threshold preference order

F.T.Aleskerov, V.I.Yakuba. A method for threshold aggregation of three-grade rankings. *Doklady Math.* **75** (2007) 322–324.

Notation: For $x = (x_1, \dots, x_n) \in X = \{1, 2, 3\}^n$ we denote by

$$v_k(x) = \text{card}\{i : 1 \le i \le n \text{ and } x_i = k\} \qquad (k = 1, 2, 3)$$

the multiplicity of grade k in the vector-alternative x. E.g., for $x = (1, 1, 1, 1, 3)$, we have: $v_1(x) = 4$, $v_2(x) = 0$ and $v_3(x) = 1$.

$$\boxed{v_1(x) + v_2(x) + v_3(x) = n \text{ and } S(x) = v_1(x) + 2v_2(x) + 3v_3(x).}$$

Definition (Aleskerov, Yakuba)**:** Given $x, y \in X$, we say that $x \succ_v y$ (x *is threshold preferred to* y) if

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Threshold preference order (continued)

N.B.: $x \approx_V y$ iff $v_1(x) = v_1(y)$, $v_2(x) = v_2(y)$ and $v_3(x) = v_3(y)$, i.e. a permutation of coordinates of x gives y, and vice versa.

N.B.: V is the restriction of the leximin from $\mathbb{R}^n$ to $X = \{1, 2, 3\}^n$.

Example. Let $n = 5$ and $x = (x_1, \dots, x_5)_N$ be a representative of the indifference class with $x_1 \leq \cdots \leq x_5$. We have the ordering in ascending V-preference:

$(1,1,1,1,1)_1$, $(1,1,1,1,2)_2$, $(1,1,1,1,3)_3$,
$(1,1,1,2,2)_4$, $(1,1,1,2,3)_5$, $(1,1,1,3,3)_6$,
$(1,1,2,2,2)_7$, $(1,1,2,2,3)_8$, $(1,1,2,3,3)_9$, $(1,1,3,3,3)_{10}$,
$(1,2,2,2,2)_{11}$, $(1,2,2,2,3)_{12}$, $(1,2,2,3,3)_{13}$, $(1,2,3,3,3)_{14}$,
previous line is continued here $(1,3,3,3,3)_{15}$,
$(2,2,2,2,2)_{16}$, $(2,2,2,2,3)_{17}$, $(2,2,2,3,3)_{18}$, $(2,2,3,3,3)_{19}$,
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Threshold preference order (continued)

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F. T. Aleskerov's question

According to the threshold preference order V we have:
$(2,2) \succ_V (1,3)$ for $n=2$, $(2,2,2) \succ_V (1,3,3)$ for $n=3$,
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F. T. Aleskerov's question

According to the threshold preference order V we have:
$(2,2) \succ_V (1,3)$ for $n=2$, $(2,2,2) \succ_V (1,3,3)$ for $n=3$,
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$$\Big(\underbrace{2,\dots,2}_{p}, k_1,\dots,k_{n-p}\Big) \succ_V \Big(1,\underbrace{3,\dots,3}_{p-1}, k_1,\dots,k_{n-p}\Big) \quad \forall p \geq 2.$$

Question (Aleskerov)**:** Given $n \geq 3$, is there a preference order $\succ$ on $X = \{1,2,3\}^n$ with the following properties:

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Outline

1. Preference orders
   - Borda and threshold preference orders
   - Superposition of preference orders

2. Results
   - Axiomatics of utility functions for $B * V$
   - The enumerating utility function

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In order to answer Aleskerov's question, we recall the notion of the superposition of two preference orders P and Q on X.

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$$P * Q = P \cup (I_P \cap Q) \qquad \text{(in this order!)}.$$

Thus, $x \succ_{P*Q} y$ iff either $x \succ_P y$, or $x \approx_P y$ and $x \succ_Q y$.

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Answer to Aleskerov's question: the order $\succ$ is the superposition of the Borda and threshold preference orders $B * V$.

Note that $V = V_1 * V_2$, where $x \succ_{V_k} y$ iff $v_k(x) < v_k(y)$ $(k = 1, 2)$.

Thus, we have $B * V = B \cup (I_B \cap V) = B * V_1$, or

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Ordering $\{1, 2, 3\}^n$ in ascending $B * V$-preference

Example. Let $n = 5$ and $x = (x_1, \ldots, x_5)_N$ be a representative of the indifference class with $x_1 \leq \cdots \leq x_5$. The ordinal number N will be found below. We have:

$(1, 1, 1, 1, 1)_1, \ (1, 1, 1, 1, 2)_2,$ S(x)=5,6

$(1, 1, 1, \underline{1, 3})_3, \ (1, 1, 1, \underline{2, 2})_4,$ S(x)=7

$(1, 1, \mathbf{1}, 2, \mathbf{3})_5, \ (1, 1, \underline{\underline{\mathbf{2}, 2, \mathbf{2}}})_6,$ S(x)=8

$(1, 1, \underline{\underline{1, 3, 3}})_7, \ (1, 1, 2, 2, 3)_8, \ (1, 2, 2, 2, 2)_9,$ S(x)=9

$(1, 1, 2, 3, 3)_{10}, \ (1, 2, 2, 2, 3)_{11}, \ (2, 2, 2, 2, 2)_{12},$ S(x)=10

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Outline

1. Preference orders
   - Borda and threshold preference orders
   - Superposition of preference orders
2. Results
   - Axiomatics of utility functions for $B * V$
   - The enumerating utility function

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Example: $F(x) = nS(x) - v_1(x)$, $x \in X$, is a utility function.

Theorem (Chistyakov, 2014)

A function $F : X = \{1, 2, 3\}^n \to \mathbb{R}$ is a utility function for $B * V$ (that is, $B * V = P(F)$) if and only if given $x, y \in X$, the following four axioms are satisfied:

A.1: $v_1(x) = v_1(y)$ *and* $v_3(x) = v_3(y)$ *imply* $F(x) = F(y)$;

e.g., $x = (1, 1, 2, 3) \approx_{B*V} (3, 1, 1, 2) = y$

A.2: $v_1(x) + 1 = v_1(y)$ *and* $v_3(x) + 1 = v_3(y)$ *imply* $F(x) > F(y)$;

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Outline

1. Preference orders
   - Borda and threshold preference orders
   - Superposition of preference orders

2. Results
   - Axiomatics of utility functions for $B * V$
   - The enumerating utility function

Ranking alternatives (F. Hausdorff: Set Theory)

Let P be a preference order on X. Given $A \subset X$, denote by

$$c(A) = \{x \in A : y \not\succ_P x \text{ for all } y \in A\} \quad \texttt{(choice function)}$$

the set of most P-preferred alternatives x from A.

- Set $X_1' = c(X)$ (alternatives of rank 1).
- If $k \geq 2$ and disjoint $X_1', \dots, X_{k-1}' \subset X$ with $\bigcup_{i=1}^{k-1} X_i' \neq X$ are already chosen, then put $X_k' = c(X \setminus (X_1' \cup \dots \cup X_{k-1}'))$.
- We have $X = X_1' \cup \dots \cup X_K'$ (disjoint union) with $K = |X/I_P|$.
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Define the surjective function $N : X \to \{1, 2, \ldots, K\}$ by:

- given $x \in X = X_1 \cup \cdots \cup X_K$, we have $x \in X_k$ for some unique number $1 \leq k \leq K$;
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- N is a utility function for P: $x \succ_P y$ iff $N(x) > N(y)$ $(x, y \in X)$.
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Since $I_{B*V} = I_V$, for $P = B * V$ we have $K = (n+2)(n+1)/2$.
Let $[a]$ be the greatest integer, which does not exceed a.

Theorem (Chistyakov, 2014)

A function N maps $X = \{1,2,3\}^n$ onto $\{1,2,\ldots,K\}$ and is the enumerating utility function for $B * V$ on X if and only if it is given as follows: if $n \le S(x) \le 2n$, then

$$N(x) = \left[\frac{S(x)-n}{2}\right] \cdot \left[\frac{S(x)-n+1}{2}\right] + n + 1 - v_1(x),$$

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Summary

In practical problems of ranking large sets (e.g., consisting of millions of alternatives), the crucial feature is the computation of the ordinal number of an alternative in the resulting ranking. The procedure of ranking under consideration can be made more effective provided a utility function (coherent with the ranking) is found in a suitable form.

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Thank you