Using Computer Modeling for Estimating Manipulability of Social Choice Rules

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Coalitional Manipulation (Example)

Plurality Rule is considered

Number of agents	3	2	2
1-st place	а	b	С
2-nd place	b	С	b
3-rd place	С	а	а

Social choice: {a}

Number of agents	3	2	2
1-st place	а	b	<u>b</u>
2-nd place	b	С	<u>C</u>
3-rd place	С	а	а

Social choice: {b}

Framework

- Voting procedure (not dictatorial) with n agents and m>2 alternatives
- 2. Each agent has preferences (linear order)
- 3. Social choice is calculated on the basis of preferences

Manipulation problem: an agent or a group of agents deviate their preferences to obtain a better social choice

Every not dictatorial social choice rule is manipulable (Gibbard(1973), Satterthwaite(1975))

Nitzan-Kelly index

n – number of agents (voters) m - number of alternatives m! – number of different preferences (linear orders) $(m!)^n$ - total number of profiles (profile – the set of preferences of n agents) d_0 - number of profiles, where manipulation may take place

$$NK = \frac{d_0}{(m!)^n}$$

From individual to coalitional manipulability

- 1. A coalition consists of several agents with the same preferences
- 2. All members of the coalition deviate their preferences in the same way

$$NK_{coalitional \ k=l} = \frac{d_0}{(m!)^n}$$

where l – maximum number of agents in one coalition d_0 - the number of profiles where coalition of l or less agents may manipulate

Voting rules

8 Positional Voting Rules are considered

- 1. Approval q=2
- 2. Plurality
- 3. Borda's Rule
- 4. Black's Procedure
- 5. Nanson's Procedure
- 6. Threshold Rule
- 7. Hare's Procedure
- 8. Inverse Plurality Rule

Extended preferences

- Extended preferences: allow an agent to compare all possible social choices
- There are several rules to construct extended preferences.
- In this work we consider
- 1. Leximin
- 2. Leximax
- 3. Risk-lover
- 4. Risk-averse

Computer Modeling

- 1. Generate a profile (voting situation)
- 2. Consider all possible coalitions and possible attempts to manipulate
- 3. Check whether the profile is manipulable
- 4. If yes, increase d_0 by 1

We consider 1,000,000 profiles to get results for NK indices for every m and n

The problem implies lot of computation

I=2, 3 alternatives, Leximax 0.45 0.4 0.35 Value 0.3 0.25 0.2 0.15 10 20 40 70 80 100 30 50 60 90 Number of agents ★ 2-stable set 🐥 3-stable set 🔶 Minimal undominated set 🔸 Fishburn 📥 Uncovered set II

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I=2, 3 alternatives, Leximax



Results

Several months of computations 5 machines Multiple threads

m = 3,4,5 alternatives n = 3..100 agents



Thank you!

Questions?