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Multi-criteria analysis of natural gas destination in Brazil: A comparison of TODIM against the use of the Choquet integral

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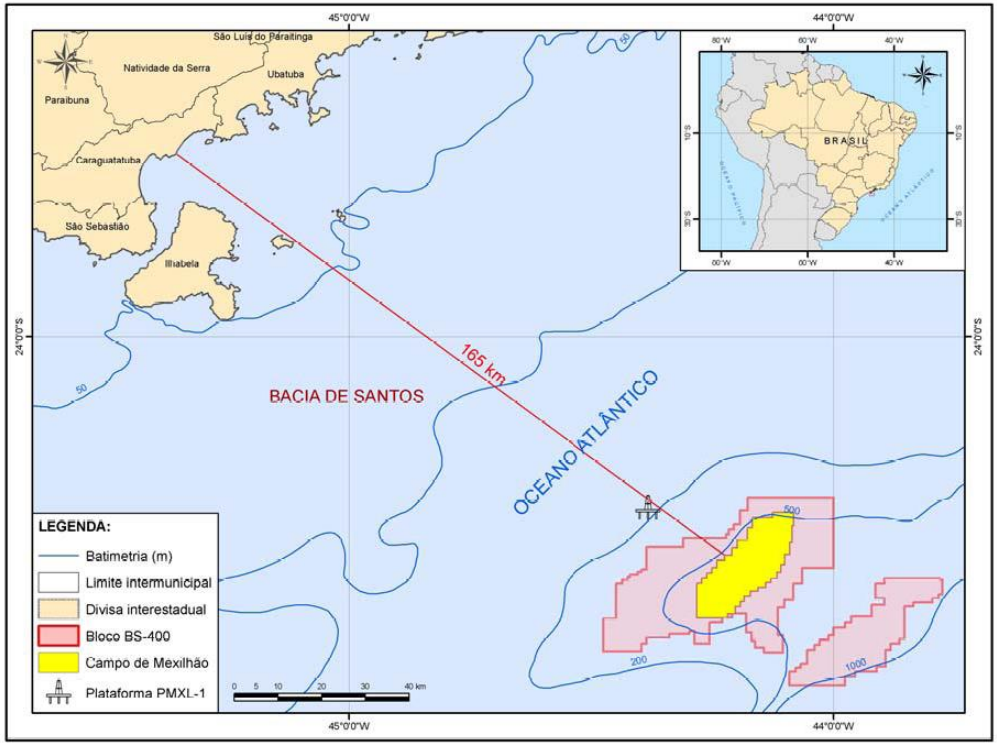
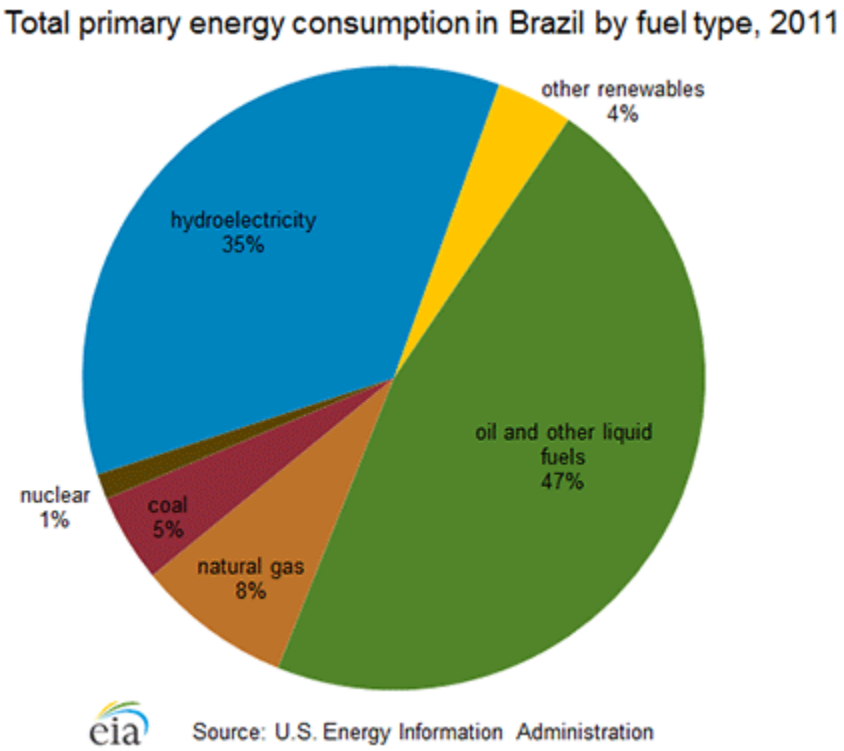
The research problem presented in this paper: selecting the best option for disposal of natural gas reserves recently discovered in the Santos Basin, more specifically in the field of Mexilhão, in Brazil. Pros and cons concerning the different alternatives are considered in the analysis. The resources of natural gas in Mexilhão are of the order of de 2.52 tcf and these to the associated oil are about 10 mmbbl.



A previous attempt to solve this problem: Gomes, L.F.A.M., Rangel, L.A.D., Maranhão, F.J.C. Multicriteria Analysis of Natural Gas Destination in Brazil: An Application of the TODIM method. Mathematical and Computer Modelling, 2009, 50, p. 92-100.


Total primary energy consumption in Brazil by fuel type, 2011
other renewables
4%
hydroelectricity
35%
oil and other liquid
fuels
47%
nuclear
1%
coal
5%
natural gas
8%
eia
Source: U.S. Energy Information Administration

In that first, 2009 attempt the TODIM method was used as a multi-criteria analytical tool. About four years later we not only updated our data basis but we also used the same method although extended by the use of the Choquet Integral – thus allowing taking into considerations measures of interactions between criteria. On such an extension see: (1) Gomes, L.F.A.M., Machado, M.A.S.; González, X.I., Rangel, L.A.D. Behavioral Multi-criteria Decision Analysis: the TODIM Method with Criteria Interactions. *Annals of Operations Research*, 2013, **211**, Issue 1, p. 531-548; and (2) Gomes L.F.A.M., Machado, M.A.S., da Costa, F.F., Rangel L.A.D. Criteria Interactions in Multiple Criteria Decision Aiding: A Choquet Formulation for the TODIM Method. *Procedia Computer Science*, 2013, **17**, p. 324-331.

The TODIM method (an acronym in Portuguese of Interactive and Multicriteria Decision Making) is based on nonlinear CPT as the shape of its value function is the same as the gains/losses function of Cumulative Prospect Theory (Tversky and Kahneman, 1992). Here gains and losses are always established with respect to a reference point. In algorithmic form an application of TODIM would follow the steps below:

Step 1: From the evaluation matrix of size m (criteria) versus n (alternatives) and criteria weights, compute values of $\Phi_c(A_i, A_j)$ by using equation (2) and making θ vary in [1,10];

Step 2: Compute values of $\delta(A_i, A_j)$ with equation (1);

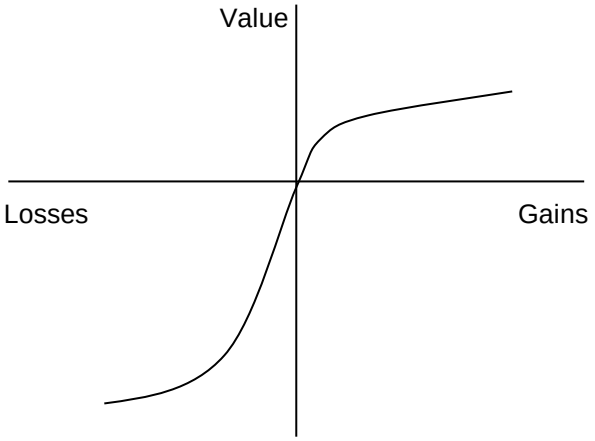
Step 3: Compute values of ξ_i with equation (3): those values lead to the ranking of alternatives.

The mathematical expressions below constitute the models underlying the use of the TODIM method:

$$\delta(A_i, A_j) = \sum_{c=1}^{m} \Phi_c(A_i, A_j) \qquad i, j = 1, ..., n$$

$$\Phi_c(A_i, A_j) = \begin{cases} \sqrt{\dfrac{w_{rc}(P_{ic} - P_{jc})}{\sum_{c=1}^{m} w_{rc}}} & if\ (P_{ic} - P_{jc}) > 0 \\ 0 & if\ (P_{ic} - P_{jc}) = 0 \\ -\dfrac{1}{\theta}\sqrt{\dfrac{(\sum_{c=1}^{m} w_{rc})(P_{jc} - P_{ic})}{w_{rc}}} & if\ (P_{ic} - P_{jc}) < 0 \end{cases}$$

Figure 1 – Value Function of the TODIM Method



Value function of the TODIM method

where:

$\delta(A_i, A_j)$ = measurement of dominance of alternative A_i over alternative A_j;
n = the total number of alternatives;
m = the total number of criteria;
c = any criterion;
w_{rc} = trade-off rate (or trade-off weighting factor) between the reference criterion r and any other, generic criterion c. The subscript r identifies a reference criterion for the decision maker. That can be, for example, the criterion that the decision maker considers as the most important one. It is easy to see that any criterion can be chosen as the reference criterion and this particular choice does not influence the final results from the computations;
P_{ic}, P_{jc} = evaluations of alternatives i and j with respect to criterion c;
θ = attenuation factor of the losses; different choices of θ lead to different shapes of the prospect theoretical value function in the negative quadrant;
$\Phi_c(A_i, A_j)$ = contribution of criterion c to function $\delta(A_i, A_j)$, when comparing alternatives A_i and A_j.
ξ_i = normalized global performance of alternative A_i, when compared against all other alternatives.

$$\xi_i = \frac{\sum_{j=1}^{n} \delta(A_i, A_j) - \min_i \sum_{j=1}^{n} \delta(A_i, A_j)}{\max_i \sum_{j=1}^{n} \delta(A_i, A_j) - \min_i \sum_{j=1}^{n} \delta(A_i, A_j)}, \quad (i = 1,\ 2,\ 3,\ ...,n)$$

A number of extensions of the original CPT-based TODIM method are available in the literature today, such as:

- TODIM with interval data, by Fa-Dong et al. (2010)
- Fuzzy TODIM, by Krohling and Souza (2012)
- Fuzzy TODIM for Group Decision Making, by Souza and Krohling (2012)
- TODIM-FSE (from Fuzzy Synthetic Evaluation), or Classificatory TODIM, by Passos et al. (2013)
- Intuitionistic fuzzy TODIM, by Krohling et al. (2013)
- TODIM for hybrid data, by Fan et al. (2013)
- Intuitionistic fuzzy TODIM for Group Decision Making, by Krohling and Pacheco (2013)
- TODIM with the Hellinger distance, by Lourenzutti and Krohling (2014)
- TODIM for hesitant fuzzy environments, by Zhang and Xu (2014)

Through considering the fuzzy measures μ of interactions between criteria we can obtain the overall value of each alternative with no need of normalization. This is accomplished by rewriting the equation that gives the measure of dominance as follows:

$$\delta(\mathrm{A_i}, \mathrm{A_j}) = \sum_{c=1}^{m} I(a)_{\mu_c} \Phi_c(A_i, A_j), \qquad \forall(\mathrm{A_i}, \mathrm{A_j})$$

where I is the Choquet Integral with respect to the fuzzy measure μ.

We can now compute the performance matrix by taking into consideration the fuzzy measures!

Performance matrix with fuzzy measures

Criteria	Alternatives			
	A_1	A_2	…	A_n
C_1	$\mu_1\Phi(A_1,C_1)$	$\mu_1\Phi(A_2,C_1)$	…	$\mu_1\Phi(A_n,C_1)$
C_2	$\mu_{12}\Phi(A_1,C_2)$	$\mu_{12}\Phi(A_2,C_2)$	. . .	$\mu_{12}\Phi(A_n,C_2)$
…	…	…	…	…
C_m	$\mu_{m-1,m}\Phi(A_1,C_m)$	$\mu_{m-1,m-1}\Phi(A_1,C_m)$	. . .	$\mu_{m-1,m-1}\Phi(A_n,C_m)$

Our problem is to rank 6 strategic alternatives under 8 evaluation criteria by using both the original TODIM method – withough explicitly considering measures of interactions between criteria – and the TODIM method extended by the use of the Choquet Integral.

Alternatives

A_1_ accelerate development for the domestic market without Bolivia (acceleration of the development of the reserve, with the domestic market as the exclusive destination and without the expansion of the Bolivia-Brazil gas pipeline);
A_2_ accelerate development for the domestic market with Bolivia(acceleration of the development of the reserve, with the domestic market as the exclusive destination with the expansion of the Bolivia-Brazil gas pipeline);
A_3_ accelerate development for the domestic market and exportation (LNG) with Bolivia (acceleration of the development of the reserve, with the domestic market as the partial destination and exportation via the LNG (liquified natural gas) with the expansion of the Bolivia-Brazil gas pipeline);
A_4_ normal development for the domestic market without Bolivia(normal development of the reserve with the domestic market as the exclusive destination and without the expansion of the Bolivia-Brazil gas pipeline);
A_5_ normal development for the domestic market with Bolivia (normal development of the reserve with the domestic market as the exclusive destination with the expansion of the Bolivia-Brazil gas pipeline);
A_6_ normal development for the domestic market and exportation (LNG) with Bolivia (normal development of the reserve with the domestic market as the partial destination and exportation via LNG with the expansion of the Bolivia-Brazil gas pipeline);
A_7_ option dummy best case fictitious option which has the best graduation in each of the criteria interpreted;
A_8_ option dummy worst case fictitious option which has the worst graduation in each of the criteria interpreted.

Criteria

C_1_return vs. risk;
C_2_social and environmental impact of the alternative;
C_3_technology available;
C_4_general regulation (tax, HSSE, price, market);
C_5_political aspects;
C_6_alignment with company strategy;
C_7_demand vs. National supply balance;
C_8_timing of implementation of the option.

Judgements were provided by a panel of 5 experts from the field of Oil & Gas. The following two rankings were thus obtained:

Alternatives	Choquet-extended TODIM Ranking	Original TODIM Ranking	Comments
A_1	2	2	same
A_2	6	6	same
A_3	5	7	-------
A_4	3	3	same
A_5	7	5	-------
A_6	4	4	same
A_7	1	1	same
A_8	8	8	same

A sensitivity analysis was performed by modifying the fuzzy measures by increasing and then decreasing their values; the Choquet Integral was recomputed and no result was significantly modified. In order to establish whether there are differences between the two methods the non-parametric sign test was used; the null hypothesis was that there are no statistically significant differences between the two methods and the alternative hypothesis was that there were differences, with 95% of confidence. The criterion for rejection of the null hypothesis was that the value of the test statistic (p-value) was less than 5%.

The table below shows that there are no differences between the results from the two methods.

Pair of Variables	Sign Test, tests are significant at p <0.05			
	No. tested	Percent	Z	p-level
TODIM & Choquet-extended TODIM	8	0.00	2.765	0.304

Main conclusions

(a) The use of the Choquet Integral minimizes the calculations of the TODIM method since it is unnecessary to normalize the raw data.

(b) By using the Choquet Integral more complex additive models can be used that allow for taking dependencies between criteria into consideration.

Suggestions for future research

(a) Tackling situations where input data on preferences are either entirely unavailable or only partially available and the decision analyst still wants to use TODIM for providing a framework on which an analysis can be based. This case can then be treated as in inverse problem and therefore approached by Monte Carlo simulation. This will lead to a SMAA-P type of method.

(b) Using more complex additive models that allow for taking dependencies between criteria into consideration;

(c) Making use of both Mamdani's and Sugeno's fuzzy inferential systems in order to compare the obtained results against these computed by the Choquet-extend TODIM method.

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Спасибо большое!

Thank you very much!