Interval-Valued Intuitionistic Fuzzy TODIM

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Summary

1. Interval-Valued Intuitionistic Fuzzy
2. Interval-Valued Intuitionistic Fuzzy Multi-criteria Decision Making
3. Interval-Valued Intuitionistic Fuzzy TODIM
4. Simulation Results
5. Conclusions

1. Interval-Valued Intuitionistic Fuzzy

- Let X be a non-empty universe of discourse, then an interval-valued intuitionistic fuzzy set (IVIFS) Ã over X is defined by:

$$\tilde{A} = \left\{ \left\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) \right\rangle \middle| x \in X \right\},$$

$$\mu_{\tilde{A}} : X \to [0,\ 1]\ \nu_{\tilde{A}} : X \to [0,\ 1]$$

- The numbers $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ stands for the degree of membership and non-membership of *x* in Ã, respectively, with the conditions:

$$0 \le \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \le 1\ \middle| \forall x \in X.$$

Each $x \in X$, $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ are closed intervals and their lower and upper bounds are denoted by $\mu_{\tilde{A}}^{L}(x), \mu_{\tilde{A}}^{U}(x), \nu_{\tilde{A}}^{L}(x), \nu_{\tilde{A}}^{U}(x)$

Therefore $\to \tilde{A} = \left\{ \left\langle x, [\mu_{\tilde{A}}^{L}(x),\ \mu_{\tilde{A}}^{U}(x)], [\nu_{\tilde{A}}^{L}(x),\ \nu_{\tilde{A}}^{U}(x)] \right\rangle \middle| x \in X \right\},$

Let two IVIFN $\tilde{a} = ([a_1, a_2], [a_3, a_4])$ and $\tilde{b} = ([0.2, 0.5], [0.3, 0.4]),$

then the distance between them is calculated by

$$d(\tilde{a}, \tilde{b}) = \frac{1}{4}\left[|a_1 - b_1| + |a_2 - b_2| + |a_3 - b_3| + |a_4 - b_4| \right]^{1/2}$$

2. Interval-Value Intuitionistic Fuzzy Multi-criteria Decision Making

- Let us consider the fuzzy decision matrix A, which consists of alternatives and criteria, described by:

$$A = \begin{array}{c} \\ A_1 \\ \ldots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & \ldots & C_n \end{array} \\ \begin{pmatrix} \tilde{x}_{11} & \cdots & \tilde{x}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \cdots & \tilde{x}_{mn} \end{pmatrix} \end{array}$$

- Where $A_1, A_2, \cdots, A_m$ are alternatives, $C_1, C_2, \ldots, C_n$ the values $\tilde{x}_{ij}$ are interval-valued intuitionistic fuzzy numbers that indicates the rating of the alternative A_i with respect to criterion C_j

- The weight vector $W = (w_1, w_2 \ldots, w_n)$ composed of the individual weights for each criterion satisfying:

$$\sum_{j=1}^{n} w_j = 1.$$

3. The TODIM method

Step 1: Normalization of the decision matrix

Step 2: Calculate the dominance among alternatives

$$\delta(R_i, R_j) = \sum_{c=1}^{m} \phi_c(R_i, R_j) \qquad \forall (i, j)$$

where

$$\phi_c(R_i, R_j) = \begin{cases} \sqrt{\dfrac{w_{rc}(r_{ic} - r_{jc})}{\sum_{c=1}^{m} w_{rc}}} & \text{if } (r_{ic} > r_{jc}) \\ 0, & \text{if } (r_{ic} = r_{jc}) \\ \dfrac{-1}{\theta}\sqrt{\dfrac{\left(\sum_{c=1}^{m} w_{rc}\right)(r_{ic} - r_{jc})}{w_{rc}}} & \text{if } (r_{ic} < r_{jc}) \end{cases}$$

Step 3: Calculate the final value

$$\xi_i = \frac{\sum \delta(i, j) - \min \sum \delta(i, j)}{\max \sum \delta(i, j) - \min \sum \delta(i, j)}$$

3. Interval-Valued Intuitionistic Fuzzy TODIM

- The interval-valued intuitionistic fuzzy TODIM is described in the following steps:

  1) Normalize the interval-valued intuitionistic fuzzy decision matrix with $\tilde{A} = \left[\tilde{x}_{ij}\right]_{mxn}$ with $\tilde{x}_{ij} = \left[a_{ij}^{L},\ a_{ij}^{U}\right],\left[b_{ij}^{L},\ b_{ij}^{U}\right]$ into the interval-valued intuitionistic fuzzy decision matrix $\tilde{R} = \left[\tilde{r}_{ij}\right]_{mxn}$ with

  $\tilde{r}_{ij} = \left[\mu_{ij}^{L},\ \mu_{ij}^{U}\right],\left[\nu_{ij}^{L},\ \nu_{ij}^{U}\right]$ using the following expressions:

$$\mu_{ij}^{L} = \frac{a_{ij}^{L}}{\left(\sum_{k=1}^{m}\left((a_{kj}^{L})^{2} + (a_{kj}^{U})^{2}\right)\right)^{\frac{1}{2}}} \text{ and } \mu_{ij}^{U} = \frac{a_{ij}^{U}}{\left(\sum_{k=1}^{m}\left((a_{kj}^{L})^{2} + (a_{kj}^{U})^{2}\right)\right)^{\frac{1}{2}}} \text{ with } i = 1,\ldots,m;\ j = 1,\ \ldots n,$$

$$\nu_{ij}^{L} = \frac{b_{ij}^{L}}{\left(\sum_{k=1}^{m}\left((b_{kj}^{L})^{2} + (b_{kj}^{U})^{2}\right)\right)^{\frac{1}{2}}} \text{ and } \nu_{ij}^{U} = \frac{b_{ij}^{U}}{\left(\sum_{k=1}^{m}\left((b_{kj}^{L})^{2} + (b_{kj}^{U})^{2}\right)\right)^{\frac{1}{2}}} \text{ with } i = 1,\ldots,m;\ j = 1,\ \ldots n,$$

3. Interval-Valued Intuitionistic Fuzzy TODIM

2) Calculate the dominance of each alternative $\tilde{R}_i$ over each alternative $\tilde{R}_j$ using the following expression:

$$\delta(\tilde{R}_i, \tilde{R}_j) = \sum_{c=1}^{m} \phi_c(\tilde{R}_i, \tilde{R}_j) \qquad \forall (i, j)$$

where:

$$\phi_c(\tilde{R}_i, \tilde{R}_j) = \begin{cases} \sqrt{\dfrac{w_{rc}}{\sum_{c=1}^{m} w_{rc}}} \cdot d(\tilde{r}_{ic}, \tilde{r}_{jc}) & \text{if } (\tilde{r}_{ic} > \tilde{r}_{jc}) \\ 0, & \text{if } (\tilde{r}_{ic} = \tilde{r}_{jc}) \\ \dfrac{-1}{\theta} \sqrt{\dfrac{\left(\sum_{c=1}^{m} w_{rc}\right)}{w_{rc}}} \cdot d(\tilde{r}_{ic}, \tilde{r}_{jc}) & \text{if } (\tilde{r}_{ic} < \tilde{r}_{jc}) \end{cases}$$

3) Calculate the global value of the alternative *i* by

$$\xi_i = \frac{\sum \delta(i, j) - \min \sum \delta(i, j)}{\max \sum \delta(i, j) - \min \sum \delta(i, j)}$$

4. Simulation Results

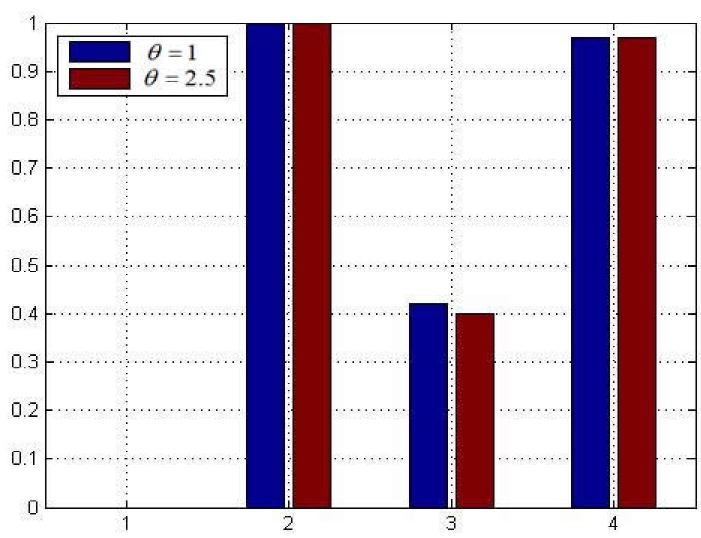
- The decision making problem investigated by Nayagam, Muralikrishnan, and Sivaraman [10] is used as benchmark.
- There are four alternatives to invest the money: A_1 is a car company, A_2 is a food company, A_3 is a computer company, and A_4 is an arms company
- The alternatives are evaluated according to three criteria: C_1 is the risk analysis, C_2 is the growth analysis, and C_3 is the environmental impact analysis.
- The weight vector associated to each criterion is $W = (w_1, w_2, w_3, w_4) = (0.35, 0.25, 0.3, 0.40)$
- The factor of attenuation of losses θ, was set to $\theta = 1$ but the value $\theta = 2.5$ has also been used.

4. Simulation Results

- Interval-valued intuitionistic fuzzy decision matrix

$$\begin{pmatrix} ([0.4,0.5],[0.3,0.4]) & ([0.4,0.6],[0.2,0.4]) & ([0.1,0.3],[0.5,0.6]) \\ ([0.6,0.7],[0.2,0.3]) & ([0.6,0.7],[0.2,0.3]) & ([0.4,0.8],[0.1,0.2]) \\ ([0.3,0.6],[0.3,0.4]) & ([0.5,0.6],[0.3,0.4]) & ([0.4,0.5],[0.1,0.3]) \\ ([0.7,0.8],[0.1,0.2]) & ([0.6,0.7],[0.1,0.3]) & ([0.3,0.4],[0.1,0.2]) \end{pmatrix}$$

- Ranking of the alternatives



- The order of the alternatives obtained is:

$$A_2 \succ A_4 \succ A_3 \succ A_1$$

- is the same as compared with that reported by Nayagam, Muralikrishnan, and Sivaraman [10]

5. Conclusions

- The interval-valued intuitionistic fuzzy TODIM method presented is able to tackle MCDM problems affected by uncertainty represented by interval-valued intuitionistic fuzzy numbers
- Interval-valued intuitionistic fuzzy numbers is a much more natural way to describe rating of the alternatives
- The IVIF-TODIM method has been investigated on two examples. In both cases, simulation results demonstrate the effectiveness of the presented method
- Applications of the proposed method to other challenging MCDM problems are under investigation

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Complete list of references cited in the paper

Thank you for your attention

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