An analysis of several novel frameworks and models in the consensus reaching process

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Outline

- **Background: consensus reaching process**
- **Model I: consensus with minimum adjustments**
- **Model II: consensus based on consistency and consensus measures**
- **Model III: direct consensus framework**
- **Future research**

Background: consensus reaching process

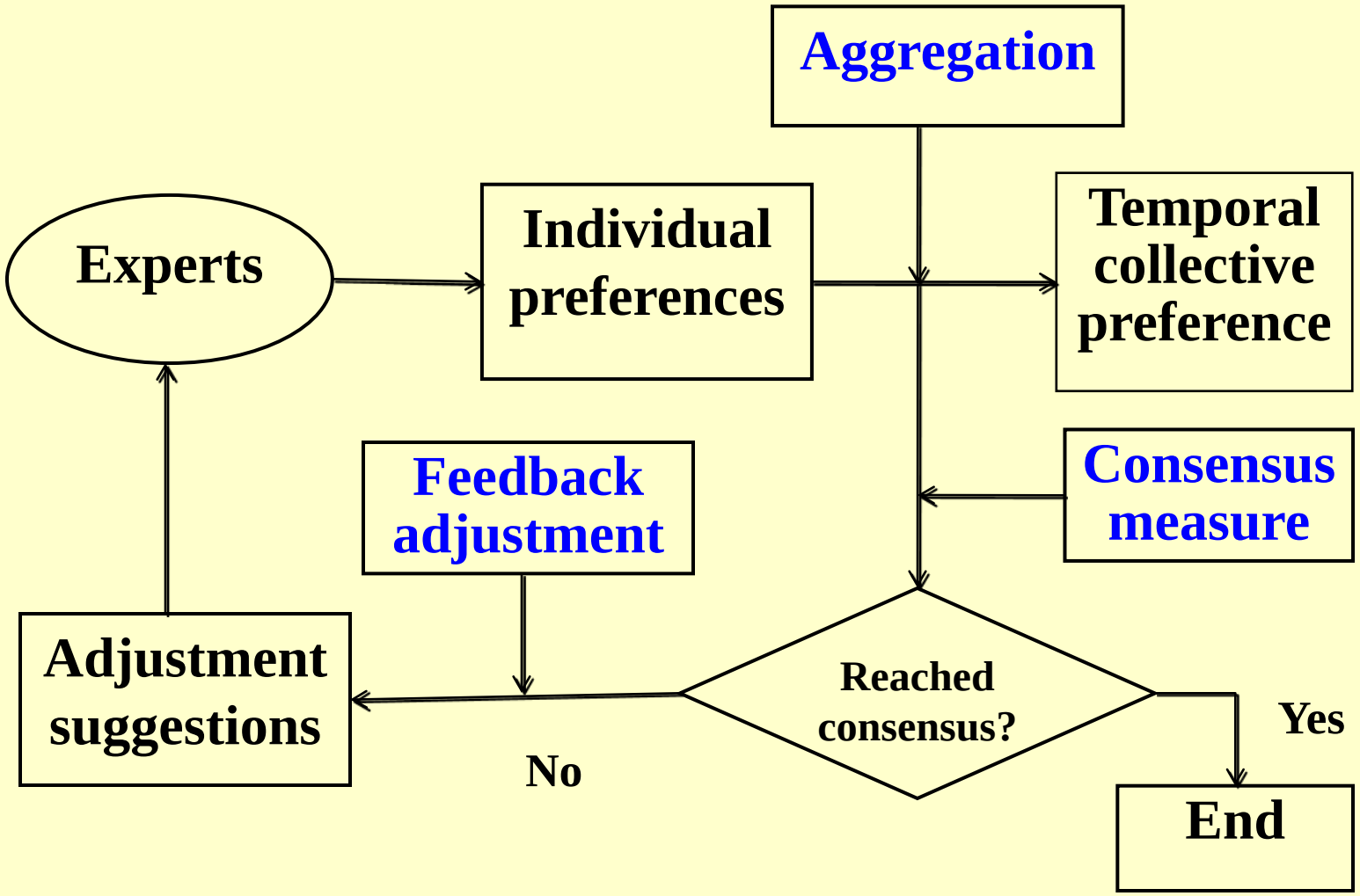
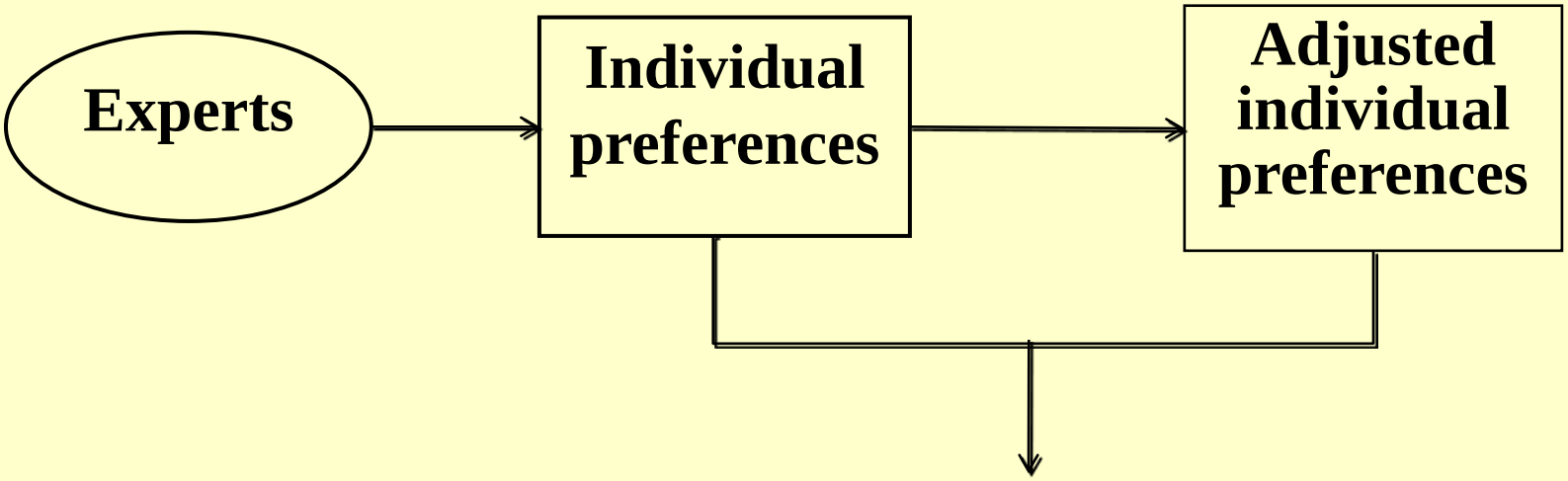


Fig. 1. A common consensus framework

Consensus with minimum adjustments



Motivation: How to minimize the adjustments?

Consensus with minimum adjustments

- **Minimizing the distance between the original and adjusted preferences**

- **Minimizing the number of adjusted preference values**

Minimizing the distance between the original and adjusted preferences

Basic model

$$\min_{\bar{o}_k} \sum_{k=1}^{m} |\bar{o}_k - o_k|$$

$$s.t. \begin{cases} \bar{o} = Ag(\bar{o}_1, \bar{o}_2, ..., \bar{o}_m) \\ |\bar{o}_k - \bar{o}| \leq \alpha, \ k = 1, 2, ..., m \end{cases} \quad (1)$$

[1] G.Q. Zhang, Y.C. Dong, Y.F. YF, H.Y. Li. Minimum-cost consensus models under aggregation operators. ***IEEE Trans. Syst., Man, Cybern. A, Syst. Humans.,*** 41 (2011) 1253-1261.

[2] B.W. Zhang, Y.C. Dong, Y.F. Xu. Maximum expert consensus models with linear cost function and aggregation operators. ***Comput. Ind. Eng.,*** 66 (2013) 147-157.

Minimizing the number of adjusted preference values

Basic model

$$\min \sum_{k=1}^{m} co_k$$

$$s.t.\begin{cases} \bar{o} = Ag(\bar{o}_1, \bar{o}_2, \ldots, \bar{o}_m) \\ \sum_{k=1}^{m} (|\bar{o}_k - \bar{o}|) \le \alpha \\ co_k = \begin{cases} 1, & o_k \ne \bar{o}_k \\ 0, & o_k = \bar{o}_k \end{cases} \end{cases} \qquad (2)$$

[1] B.W. Zhang, Y.C. Dong, Y.F. Xu. Multiple attribute consensus rules with minimum adjustments to support consensus reaching. Submitted to ***Knowl.-Based Syst.***

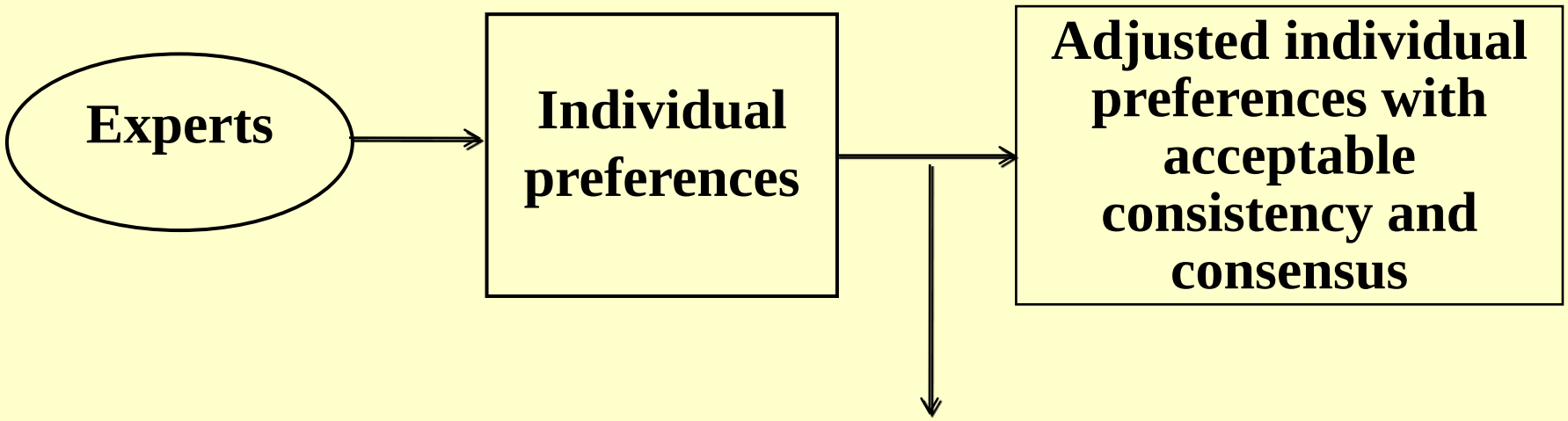
Consensus with minimum adjustments

➢Advantages

- Minimize adjustment amounts.

- Provide new references for experts to modify their preferences.

Consensus based on consistency and consensus measures



Motivation: How to preserve/improve individual consistency in consensus reaching process?

Consensus based on consistency and consensus measures

- **Iteration-based consensus model**

  (Take multiplicative preference relations as example)

- **Optimization-based consensus model**

  (Take additive preference relations as example)

Iteration-based consensus model

➢ Consistency and consensus measures

- The *consistency index* of A^k is defined by

$$\overline{CI}(A^k) = \frac{2}{(n-1)(n-2)} \sum_{i<j} (\log(a_{ij}^k) - \log(p_i^k) + \log(p_j^k))^2$$

- The *consensus index* of A^k is defined by

$$CI(A^k) = \frac{2}{(n-1)(n-2)} \sum_{i<j} (\log(a_{ij}^k) - \log(p_i^c) + \log(p_j^c))^2$$

Iteration-based consensus model

Feedback adjustment

When constructing $\overline{A}^k = [\overline{a}_{ij}^k]_{n\times n}$, we suggest that $\overline{a}_{ij}^k = (a_{ij}^k)^{\theta}(p_i^c / p_j^c)^{(1-\theta)}$, where $0 < \theta < 1$.

Iteration-based consensus model

➢ Advantages

- The consistency level of multiplicative preference relations are improved.
- The consensus level is improved.
- The Pareto principle is satisfied.

[1] Y.C. Dong, G.Q. Zhang, W.C. Hong, Y.F. Xu. Consensus models for AHP group decision making under row geometric mean prioritization method. ***Deci. Support Syst.*** 49 (2010) 281-289.

Optimization-based consensus model

➢Basic model

$$\min_{\overline{F}^k} \sum_{k=1}^{m} d(F^k, \overline{F}^k)$$

$$s.t. \begin{cases} \overline{CI}(\overline{F}^k) \le \beta, \quad k = 1, 2, ..., m \\ CI\{\overline{F}^1, ..., \overline{F}^m\} \le \alpha \end{cases} \qquad (3)$$

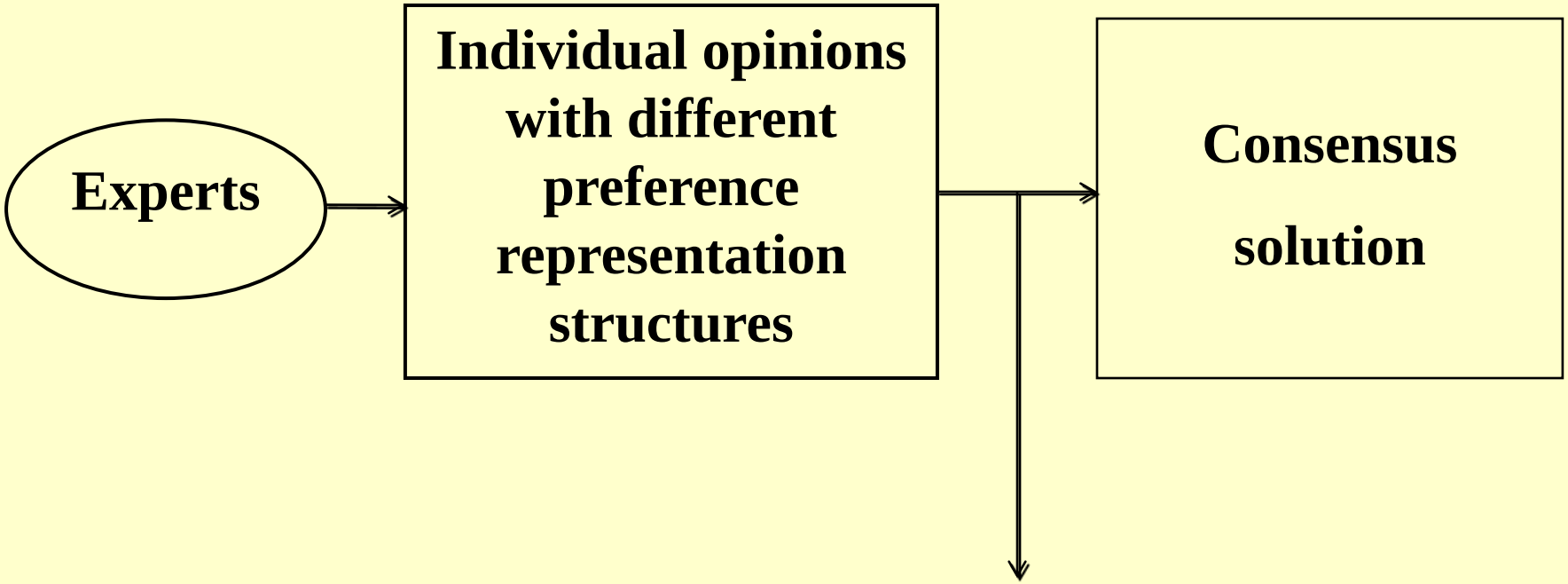
[1] G.Q. Zhang, Y.C. Dong, Y.F. Xu. Linear optimization modeling of consistency issues in group decision making based on fuzzy preference relations. ***Expert Syst. Appl*** **.** 39 (2012) 2415-2420.

Optimization-based consensus model

➢Advantages

- The consistency and consensus are improved in one linear programming model.
- Individuals' original preferences are preserved as much as possible.

Direct consensus framework



Motivation: How to obtain the consensus solution?

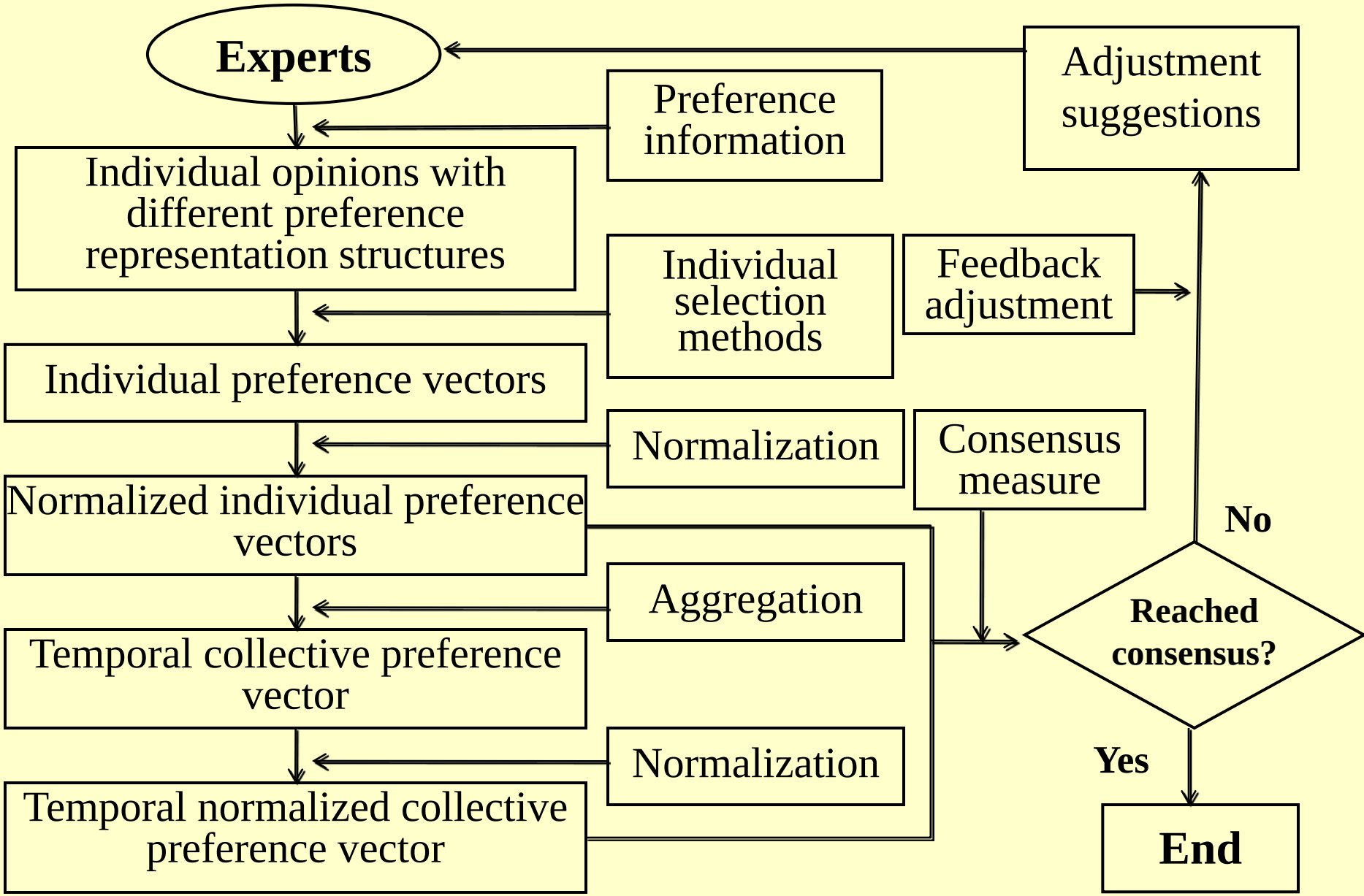


Fig. 2. Direct consensus framework

Direct consensus framework

➢ Advantages

- The internal inconsistency issue is avoided.
- The Pareto principle is satisfied.

[1] Y.C. Dong, H.J. Zhang. Multiperson decision making with different preference representation structures: A direct consensus framework and its properties. ***Knowl.-Based Syst.*** 58 (2014) 45-57.

Future research

- Incorporate the behaviours of experts into the consensus reaching process.
- Investigate the consensus reaching process in dynamic situations.
- Design a general comparison framework to compare different consensus models.

Thank you!

Any questions?